## 19 The Laws of Motion

This chapter pulls together some basic physics ideas that are used in several places in the book.
We will pay special attention to rotary motion, since it is less familiar to most people than ordinary straight-line motion. Gyroscopes, in particular, behave very differently from ordinary non-spinning objects. It is amazing how strong the gyroscopic effects can be.

### 19.1 Straight-Line Motion

First, let's review the physical laws that govern straight-line motion. Although the main ideas go back to Galileo, we speak of Newton's laws, because he generalized the ideas and codified the laws.

The first law of motion states: "A body at rest tends to remain at rest, while a body in motion tends to remain in motion in a straight line unless it is subjected to an outside force". Although that may not sound like a very deep idea, it is one of the most revolutionary statements in the history of science. Before Galileo's time, people omitted frictional forces from their calculations. They considered friction "natural" and ubiquitous, not requiring explanation; if an object continued in steady motion, the force required to overcome friction was the only thing that required explanation. Galileo and Newton changed the viewpoint. Absence of friction is now considered the "natural" state, and frictional forces must be explained and accounted for just like any others.

The second law of motion says that if there is any change in the velocity of an object, the force required $(F)$ is proportional to the mass $(m)$ of the object, and proportional to the acceleration vector (a). In symbols,

$$
F=m a
$$

(19.1)

The acceleration vector is defined to be the rate-of-change of velocity. See below for more on this.
The following restatement of the second law is often useful: since momentum is defined to be mass times velocity, and since the mass is not supposed to be changing, we conclude that the force is equal to the rate-of-change of the momentum. To put it the other way, change in momentum is force times time.

The third law of motion says that if a force is applied to an object, an equal and opposite force must be applied somewhere else. This, too, can be restated in terms of momentum: if we impart a certain momentum to an object, we must impart an equal and opposite amount of momentum to something else. ${ }^{1}$ This means that the total momentum of the world cannot change. This principle --- conservation of momentum --- is one of the most fundamental principles of physics, on a par with the conservation of energy discussed in chapter 1.

## * Two Notions of Acceleration

The quantity $a=F / m$ that appears in equation 19.1 was carefully named the acceleration vector. Care was required, because there is another, conflicting notion of acceleration:

- The scalar notion of acceleration generally means an increase in speed. It is the opposite of deceleration.
- The vector notion of acceleration is what appears in equation 19.1. It is the rate-of-change of velocity. A forward acceleration increases speed. A rearward acceleration decreases speed, but it is still called an acceleration vector. A sideways acceleration leaves the speed
unchanged, but it is still an acceleration vector, because it changes the direction of the velocity vector. There is no corresponding notion of deceleration, because any change in velocity is called an acceleration vector.

Alas, everyone uses both of these conflicting notions, usally calling both of them "the" acceleration. It is sometimes a struggle to figure out which meaning is intended. One thing is clear, though: the quantity $a=F / m$ that appears in the second law of motion is a vector, namely the rate-of-change of velocity.

Do not confuse velocity with speed. Velocity is a vector, with magnitude and direction. Speed is the magnitude of the velocity vector. Speed is not a vector.

Suppose you are in a steady turn, and your copilot asks whether you are accelerating. It's ambiguous. You are not speeding up, so no, there is no scalar acceleration. But the direction of the velocity vector is changing, so yes, there is a very significant vector acceleration, directed sideways toward the inside of the turn.

If you wish, you can think of the scalar acceleration as one component of the vector acceleration, namely the projection in the forward direction.

Try to avoid using ambiguous terms such as "the" acceleration. Suggestion: often it helps to say "speeding up" rather than talking about scalar acceleration.

## * Force is Not Motion

As simple as these laws are, they are widely misunderstood. For example, there is a widespread misconception that an airplane in a steady climb requires increased upward force and a steady descent requires reduced upward force. ${ }^{?}$ Remember, lift is a force, and any unbalanced force would cause an acceleration, not steady flight.

In unaccelerated flight (including steady climbs and steady descents), the upward forces (mainly lift) must balance the downward forces (mainly gravity). If the airplane had an unbalanced upward force, it would not climb at a steady rate --- it would accelerate upwards with an ever-increasing vertical speed.

Of course, during the transition from level flight to a steady climb an unbalanced vertical force must be applied momentarily, but the force is rather small. A climb rate of 500 fpm corresponds to a vertical velocity component of only 5 knots, so there is not much momentum in the vertical direction. The kinetic energy of ordinary (non-aerobatic) vertical motion is negligible.

In any case, once a steady climb is established, all the forces are in balance.

### 19.2 Sitting in a Rotating Frame

If we measure motion relative to a rotating observer, Newton's laws do not apply. In this section and the next, we will use what we know about non-rotating reference frames to deduce the correct laws for rotating frames.

Suppose Moe is riding on a turntable; that is, a large, flat, smooth, horizontal rotating disk, as shown in figure 19.1. Moe has painted an $X, Y$ grid on the turntable, so he can easily measure positions,
velocities, and accelerations relative to the rotating coordinate system. His friend Joe is nearby, observing Moe's adventures and measuring things relative to a nonrotating coordinate system.


Figure 19.1: Rotating and Non-Rotating Coordinate Systems
We will assume that friction between the puck and the turntable is negligible.
The two observers analyze the same situation in different ways:

Moe immediately observes that Newton's first law does not apply in rotating reference frames.

An unattached hockey puck initially at rest with respect to the turntable (anywhere except right at the center) does not remain at rest; it accelerates outwards. This is called centrifugal acceleration.

To oppose the centrifugal acceleration, Moe holds the puck in place with a rubber band, which runs horizontally from the puck to an attachment point on the turntable. By measuring how much the rubber band stretches, Moe can determine the magnitude of the force.

Moe says the puck is not moving relative to his reference frame. The rubber band compensates for the centrifugal force.

In Joe's nonrotating frame, Newton's laws do apply.

In a nonrotating frame, there is no such thing as centrifugal acceleration. The puck moves in a straight line, maintaining its initial velocity, as shown in figure 19.2.

Joe can observe the same rubber band. Moe and Joe agree about the magnitude and direction of the force.

Joe says that the puck's momentum is constantly changing due to the rotation. The rubber band provides the necessary force.

### 19.3 Moving in a Rotating Frame

We now consider what happens to an object that is moving relative to a rotating reference frame.
Suppose Moe has another hockey puck, which he attaches by means of a rubber band to a tiny tractor. He drives the tractor in some arbitrary way. We watch as the puck passes various marks ( $A$, $B$, etc.) on the turntable.

Moe sees the puck move from mark $A$ to mark $B$. The marks obviously are not moving relative to his reference frame.

Joe agrees that the puck moves from mark $A$ to mark $B$, but he must account for the fact that the marks themselves are moving.

So let's see what happens when Joe analyzes the compound motion, including both the motion of the marks and the motion of the puck relative to the marks.

There are actually four contributions:

1. If the puck is accelerating relative to Moe's rotating frame, Joe agrees and counts that as a contribution to the acceleration. Both observers agree on how much force this requires.
2. From Joe's point of view, mark $A$ is not only moving; its velocity is changing. Changing this component of the puck's velocity requires a force. (From Moe's point of view, this is the force needed to oppose centrifugal acceleration, as discussed previously.)
3. The velocity of mark $B$ is different from the velocity of mark $A$. As the puck is towed along the path from point $A$ to point $B$, the rubber band must provide a force in order to change the velocity so the puck can "keep up with the Joneses".
4. The velocity of the puck relative to the marks is also a velocity, and it must also rotate as the system rotates. This change in velocity also requires a force.

We can say a few words about each of these contributions from Moe's point of view:

1. This " $F=m a$ " contribution is completely unsurprising. It is independent of position, independent of velocity, and independent of the frame's rotation rate.
2. The centrifugal contribution depends on position, but is independent of the velocity that Moe measures relative to his rotating reference frame. It is also independent of any acceleration created by Moe's tractor. It is proportional to the square of the frame's rotation rate.
3. This contribution is independent of position. It is proportional to the velocity that Moe measures, and is always perpendicular to that velocity. It is also proportional to the first power of the frame's rotation rate.
4. This contribution is also independent of position. It is also proportional to the velocity relative to the rotating frame, and is perpendicular to that velocity, and is proportional to the first power of the frame's rotation rate.

Contribution \#3 is numerically equal to contribution \#4. The total effect is just twice what you would get from either contribution separately. Together these two contributions are called the Coriolis effect. ${ }^{3}$

The Coriolis effect can be described as an acceleration (proportional to the object's velocity), and equivalently it can be described as a force (proportional to the object's momentum).

Let's consider a reference frame attached to an eastward-rotating rotating planet, such as the earth. Near the north pole, the Coriolis acceleration is always toward your right, if you are facing forward along the direction of motion. Northward motion produces a Coriolis acceleration to the east; a very real westward force is necessary to oppose it if you want to follow a straight line painted on the earth. Eastward motion produces a Coriolis acceleration to the south; a very real northward force is necessary to oppose it.

The Coriolis argument only applies to motion in the plane of rotation. Momentum in the other direction (parallel to the axis of rotation) is unaffected. In all cases the Coriolis acceleration lies in the plane of
rotation and perpendicular to the motion.
Near the equator, we have to be careful, because the plane of rotation is not horizontal. In this region, eastward motion produces a Coriolis acceleration in the upward direction, while westward motion produces a Coriolis acceleration in the downward direction. In this region, north/south motions are perpendicular to the plane of rotation and produce no Coriolis effects.

To reiterate: The Coriolis effect and the centrifugal field are two separate contributions to the story. The Coriolis effect applies only to objects that are moving relative to the rotating reference frame. The centrifugal field affects all objects in the rotating frame, whether they are moving or not.

## * Magnitude of the Effect

Suppose you are in an airplane, flying straight ahead at 120 knots along the shortest path between two points on the earth's surface. Because of the rotation of the earth, the airplane will be subject to a Coriolis acceleration of about 0.001 G . This is too small to be noticeable.

Now suppose you and a friend are standing 60 feet apart, playing catch in the back of a cargo airplane while it is performing a standard-rate turn (three degrees per second). If your friend throws you the ball at 60 mph , it will be subject to a horizontal Coriolis acceleration of more than a quarter $G$. That means the ball will be deflected sideways about $21 / 2$ feet before it gets to you --- which is enough to be quite noticeable. In normal flying, though, we don't often throw things far enough to produce large Coriolis effects.

The wind, moving relative to the rotating earth, is subject to a Coriolis acceleration that is small but steady; the cumulative effect is tremendously important, as discussed in section 20.1.

### 19.4 Centrifuges with and without Gravity

### 19.4.1 The Centrifugal Field is Real

An airplane in a turn, especially a steep turn, behaves like a centrifuge. There are profound analogies between centrifugal and gravitational fields:

The gravitational field at any given point is an acceleration. It acts on objects, producing a force in proportion to the object's mass.

The centrifugal field at any given point is also an acceleration. It, too, acts on objects, producing a force in proportion to the object's mass.

Strictly speaking, neither gravity nor centrifugity is a "force" field. Each is really an acceleration field. Of course there is a force involved, but it is always a force per unit mass, which is properly called an acceleration.

Einstein's principle of equivalence states that at any given point, the gravitational field is indistinguishable from an acceleration of the reference frame. ${ }^{4}$ In a freely-falling reference frame, such as a freely-orbiting space station, everything is weightless.

My laboratory is not a free-falling inertial frame. It is being shoved skyward as the earth pushes on its foundations. If you measure things relative to the laboratory

Similarly, the cabin of a centrifuge is clearly not an inertial frame. If you measure things relative to the cabin, you will observe centrifugal accelerations.
walls, you will observe gravitational accelerations.

From a modern-physics point of view, both gravity and centrifugity emerge as consequences of working in an accelerated frame. There is nothing wrong with doing so, provided the work is done carefully. Accounting for centrifugal effects is not much trickier than accounting for gravitational effects. When people think this can't be done, it is just because they don't know how to do it. To paraphrase Harry Emerson Fosdick:

Person saying it can't be done is likely to be interrupted by persons doing it.

For a ground-bound observer analyzing the flight of an airplane, it may be convenient to use a reference frame where gravity exists and centrifugity does not. However, the pilot and passengers usually find it convenient to use a frame that includes both gravity and centrifugity.

The centrifugal field is not crude or informal or magical. (The problem with magic is that it can explain false things just as easily as true things.) Like the gravitational field, it is a precise way of accounting for what happens when you work in a non-freely-falling reference frame.

### 19.4.2 Centrifuge

To get a better understanding of the balance of forces in a turning and/or slipping airplane, consider the centrifuge shown in figure 19.2. For the moment we will neglect the effects of gravity; imagine this centrifuge is operating in the weightless environment of a space station. We are riding inside the centrifuge cabin, which is shown in red. We have a supply of green tennis balls. At point $A$ (the southernmost point of our path) we drop a tennis ball, whereupon it flies off as a free particle. Our centrifuge continues to follow its circular path.


Figure 19.2: An Object Departing a Centrifuge
Case 1a: Consider the point of view of a bystander (not riding in the centrifuge). The dropped tennis ball moves in a straight line, according to the first law of motion. Contrary to a common misconception, the bystander does not see the ball fly radially away from the center of the centrifuge. It just continues with the purely eastward velocity it had at point $A$, moving tangentially.

Case 1b: Consider our point of view as we ride in the centrifuge. At point $A$, the tennis ball has no velocity relative to us. For the first instant, it moves along with us, but then gradually it starts moving away. We do see the ball accelerate away in the purely radial direction. The tennis ball --- like everything else in or near the centrifuge --- seems to be subjected to a centrifugal acceleration field.

Einstein's principle of equivalence guarantees that our viewpoint and the bystander's viewpoint are equally valid. The bystander says that the centrifuge cabin and its occupants accelerate away from the freely moving tennis ball, while we say that the tennis ball accelerates away from us under the influence of the centrifugal field.

There is one pitfall that must be avoided: you can't freely mix the two viewpoints. It would be a complete fallacy for the bystander to say "The folks in the cabin told me the tennis ball accelerated outward; therefore it must move to the south starting from point $A$ ". In fact, the free-flying ball does not accelerate relative to the bystander. It will not wind up even one millimeter south of point $A$. It will indeed wind up south of our centrifuge cabin, but only because we have peeled off to the north.

Case 2a: Consider from the bystander's point of view what happens to a ball that has not been released, but is just sitting on a seat in the centrifuge. The bystander sees the ball subjected to an unbalanced force, causing it to move in a non-straight path relative to the earth.

Case 2b: Consider the seated ball from the centrifuge-riders' point of view. The force on the ball exerted by the seat is just enough to cancel the force due to centrifugal acceleration, so the forces are in balance and the ball does not move.

When analyzing unsteady motion, or when trying to calculate the motion of the centrifuge itself, it is often simpler to analyze everything from the bystander's point of view, in which the centrifugal field will not appear. On the other hand, in a steady turn, is often easy and natural to use the centrifugeriders' point of view; in which all objects will be subject to centrifugal accelerations.

### 19.4.3 Centrifuge and Gravity

Now that we understand the basic idea, let's see what happens when our centrifuge operates in the normal gravitational field of the earth. This is shown in figure 19.3. When the tennis ball departs the centrifuge, it once again travels in a purely easterly direction, but this time it also accelerates downward under the influence of gravity.


Figure 19.3: An Object Departing a Centrifuge, with Gravity
Once again, from inside the cabin we observe that the tennis ball initially accelerates away in the direction exactly away from the pivot of the centrifuge. This is no coincidence; it is because the only difference between our motion and the free-particle motion comes from the force in the cable that attaches us to the pivot.
(The foregoing applies only to the initial acceleration of the dropped ball. As soon as it picks up an appreciable velocity relative to us, we need to account for Coriolis acceleration as well as centrifugal acceleration.)

Remember, the equivalence principle says that at each point in space, a gravitational field is indistinguishable from an accelerated reference frame. Therefore we need not know or care whether the tennis ball moves away from us because we are being accelerated, or because there is a gravitating planet in the vicinity, or both.

### 19.5 Centrifugal Effects in a Turning Airplane

Let's examine the forces felt by the pilot in a turning airplane. We start with a coordinated turn, as shown in figure 19.4.


Figure 19.4: Airplane in a Coordinated Turn


Figure 19.5: Airplane in a Nonturning Slip


Figure 19.6: Airplane in a Boat Turn
In figures such as this, whenever I am analysing things using the pilot's point of view, the figure will include a rectanglular "frame" with a little stick figure (the observer) standing in it. It is important to carefully specify what frame is being used, because even simple questions like "which way is down" can have answers that depend on which observer you ask. In particular, I define N-down (Newtonian down) to mean the direction straight down toward the center of the earth. In contrast, I define E-down
(effective down, or Einsteinian down) to be the direction in which a free particle departs if you drop it. In a turning airplane, the two directions are not the same.

Using your inner ear, the seat of your pants, and/or the inclinometer ball, you can tell which way is Edown. Using the natural horizon and/or the artificial horizon, you can tell which way is N-down.

In figure 19.4, assume the airplane's mass is one ton. Real gravity exerts a force of one ton, straight down toward the center of the earth. The airplane is an a $45^{\circ}$ bank, so there is one ton of centrifugal force, sideways, parallel to the earth's horizon. All in all, the wings are producing 1.41 tons of lift, angled as shown in the figure.

The lower part of figure 19.4 analyzes the forces on the inclinometer ball. Real gravity exerts a downward force on the ball, and centrifugity exerts a sideways force. The tubular race that contains the ball exerts a force perpendicular to the wall of the race (whereas the ball is free to roll in the direction along the race). The race-force balances the other forces when the ball is in the middle, confirming that this is a coordinated turn.

Next, we consider the forces on the airplane in an ordinary nonturning slip, as shown in figure 19.5. The right rudder pedal is depressed, and the port wing has been lowered just enough that the horizontal component of lift cancels the horizontal force due to the crossflow over the fuselage. The airplane is not turning. Everybody agrees there is no centrifugal field.

As a third example, we consider what happens if you make a boat turn, as shown in figure 19.6. (For more about boat turns in general, see section 8.10.) Because the airplane is turning, it and everything in it will be subjected to a centrifugal acceleration (according to the viewpoint of the centrifuge riders).

The lower part of figure 19.6 shows how the inclinometer ball responds to a boat turn. Gravity still exerts a force on the ball, straight down. Centrifugity exerts a force sideways toward the outside of the turn. The ball is subject to a force of constraint, perpendicular to the walls of the race. (It is free to roll in the other direction.) The only place in the race where this constraint is in a direction to balance the other forces is shown in the figure. The ball has been "centrifuged" toward the outside of the turn. This is a quantitative indication that the E-down direction is not perpendicular to the wings, and some force other than wing-lift is acting on the plane.

### 19.6 Angles and Rotations

### 19.6.1 Directions of Rotation: Yaw, Pitch, and Roll

Any rotation can be described by specifying the plane of rotation and the amount of rotation in that plane. (Note that in this chapter, the word "airplane" is always spelled out, using eight letters. In contrast, the word "plane" will be reserved to refer to the thin, flat abstraction you learned about in geometry class.)


Figure 19.7: Rotations: Yaw, Pitch and Roll

Three particularly simple planes of rotation are yaw, pitch, and roll, as shown in figure 19.7. If you want a really precise definition of these three planes, proceed as follows: First: The airplane has a left-right mirror symmetry, and it is natural to choose the plane of symmetry as the plane of pitch-wise rotations. Secondly: Within the symmetry plane, we somewhat-arbitrarily choose a reference vector, attached to the airplane, that corresponds to zero pitch angle. It is conventional to choose this so that level cruising flight corresponds to zero pitch. The exact choice is unimportant. The roll-wise plane is perpendicular to this vector. Thirdly: The yaw-wise plane is perpendicular to the other two planes.

Any plane of rotation -- not just the three planes shown in figure 19.7 -- can be quantified in terms of bivectors, as discussed in section 19.7.

Older books often speak in terms of the axis of rotation, as defined in figure 19.8. In the end, it comes to the same thing: for example, yaw-wise rotation is synonymous with a rotation about the $Z$ axis.

We prefer to speak in terms of the plane of rotation. This is more modern, more sophisticated, and more in accord with the way things look when you're in the cockpit: For example, in normal flight, when the airplane yaws, it is easy to picture the nose moving left or right in a horizontal plane. This is easier than thinking about the $Z$ axis.


Figure 19.8: Axes: $X, Y$ and $Z$
Beware that older books give peculiar names to some of the axes. They refer to the $Y$ axis as the lateral axis and the $X$ axis as the longitudinal axis, which are sensible enough, but then they refer to $Y$-axis stability as longitudinal stability and $X$-axis stability as lateral stability --- which seems completely reversed and causes needless confusion. Reference 16 calls the $Z$ axis the normal axis, since it is normal (i.e. perpendicular) to the other axes --- but that isn't very helpful since every one of the axes is normal to each of the others. Other references call the $Z$ axis the vertical axis, but that is very confusing since if the bank attitude or pitch attitude is not level, the $Z$ axis will not be vertical. The situation is summarized in the following table.

This Book
Older Terminology

|  |  |
| :---: | :---: |
| yaw bivector | vertical axis |
| $X Y$ plane | $Z$ axis |
| yaw-wise stability | directional stability |
| pitch bivector | lateral axis |
| $Z X$ plane | $Y$ axis |
| pitch-wise stability | longitudinal stability |
| roll bivector | longitudinal axis |
| $Y Z$ plane | $X$ axis |
| roll-wise stability | lateral stability |

### 19.6.2 Attitude: Heading, Pitch, Bank

The term attitude describes the orientation of the airplane relative to the earth. Attitude is specified in terms of three angles: heading, pitch, and bank. (These are sometimes called the Euler angles.)

Heading, pitch, and bank are intimately related to yaw, pitch, and roll. To construct a specified attitude, imagine that the airplane starts in level flight attitude with the $X$ axis pointed due north; then:

- Rotate the airplane in the yaw-wise direction by the specified heading angle. A positive angle specifies a clockwise rotation as seen from above, so that a heading of 090 degrees corresponds to pointing east and a heading of 180 degrees corresponds to pointing south.
- Rotate the airplane in the pitch-wise direction by the specified pitch attitude angle. A positive angle specifies a nose-up attitude.
- Rotate the airplane in the roll-wise direction by the specified bank attitude angle. A positive angle corresponds to clockwise as seen from the rear.

As discussed below (section 19.6.4), it is important to perform these rotations in the order specified: yaw, then pitch, then roll.

We have just seen how, given a set of angles, we can put the airplane into a specified attitude. We now consider the reverse question: given an airplane in some attitude, how do we determine the angles that describe that attitude?

Answer: just figure out what it would take to return the airplane to level northbound attitude. The rotations must be undone in the reverse of the standard order:

- First, rotate the aircraft in the roll-wise direction until the wings are level. This determines the bank attitude.
- Second, rotate the aircraft in the pitch-wise direction until the $X$ axis is level. This determines the pitch attitude. Note that this rotation is not performed in the original $Z X$ plane, but rather in the new $Z X$ plane, which is vertical as a consequence of the previous step.
- Finally, rotate the aircraft in the (new) yaw-wise direction until the nose is pointing north. This determines the heading.


### 19.6.3 Angle Terminology

The following table summarizes the various nouns and verbs that apply to angles and motions in the three principal directions:

|  | $X Y$ plane | ZX plane | YZ plane |
| :---: | :---: | :---: | :---: |
| Motion | it yaws | it pitches | it rolls |
| Angle | the heading | the pitch attitude | the bank attitude |

Here are a few more fine points of angle-related terminology:

- Saying that the airplane is "banking" or "in a bank" refers to a definite bank attitude.
- In contrast, saying that the airplane is "rolling" or "in a roll" refers to a definite rate of rotation, i.e. a changing bank angle.
- Pitch angle usually means the same thing as pitch attitude.
- Bank angle usually means the same thing as bank attitude.
- The term roll angle is a rarely-used synonym for bank angle.
- The word turn sometimes refers to a change in which way you are pointing (i.e. yaw) and sometimes refers to a change in which way you are going. For a coordinated turn, both meanings mean the same thing, but in uncoordinated flight "turn" is distressingly ambiguous.


## * Other Angles

To define the angle of attack of the fuselage, take the direction of flight (or its reciprocal, the relative wind) and project it onto the $X Z$ plane. The angle of attack is the angle between this projection and the $X$ axis or some other convenient reference.

To define the slip angle, take the direction of flight (or the relative wind) and project it onto the $X Y$ plane. The slip angle is the angle between this projection and the $X$ axis.

Some aerodynamics texts use the term sideslip angle, which is synonymous with slip angle. Beware that some pilot-training books try to draw a distinction between a forward slip and a side slip, even though the difference is imaginary, as is discussed in conjunction with figure 11.1.

### 19.6.4 Yaw Does Not Commute with Pitch

It is a fundamental fact of geometry that the result of a sequence of rotations depends on the order in which the rotations are performed.

Note that for a sequence of ordinary non-rotational movements, the ordering does not matter. That is, suppose I have two small objects that start out at the same place on a flat surface. I move one object move two feet north, and then three feet west. I move the other object the same distances in the other order: three feet west and then two feet north. Assuming there are no obstructions, both objects will arrive at the same destination. The ordering of the movements does not matter.

However, angles don't play by the same rules as distances. For instance, there are ways of changing the yaw angle (i.e., the heading) by 37 degrees (or any other amount) without ever yawing the airplane. That is:

- You can pull up into vertical flight, use the ailerons to rotate 37 degrees in the now-horizontal roll-wise plane, and then push back to level flight.
- Another way to do the same thing is to roll into a ninety-degree bank, pull on the yoke to rotate by 37 degrees in the now-horizontal pitch-wise plane, and then roll back to wings-level attitude.

If the aircraft (and its occupants) can tolerate heavy $G$ loads, such maneuvers are perfectly fine ways to make tight turns at high airspeed.

In non-aerobatic flight, a less-extreme statement applies: a rotation in a purely horizontal plane is not a pure yaw when the aircraft is not in a level attitude. For instance, suppose you are in level flight, steadily turning to the left. This is, of course, a turn in a purely horizontal plane. Further suppose that you have a nose-up pitch attitude, while still maintaining a level flight path, as could happen during slow flight. This means that the plane of yaw-wise rotations is is not exactly horizontal. You could, in principle, perform the required heading change by pitching down to level pitch attitude, performing a pure yaw, and then pitching back up, but since rotations are not commutative this is not equivalent to maintaining your pitch attitude and performing a pure yaw. Performing the required change of heading without pitching down requires mostly pure leftward yaw, but involves some rightward rollwise rotation also.

The analysis in the previous paragraph is $100 \%$ accurate, but completely irrelevant when you are piloting the airplane.- Arguing about whether the heading change is a pure yaw or a yaw plus roll is almost like arguing about whether a glass of water is half full or half empty --- the physics is the same. In this case the physics is simple: the inside (left) wing follows a horizontal circular path, while the outside (right) wing follows a slightly longer horizontal path around a larger circle.

It is easy to see why that is so: The turn requires a rotation in a horizontal plane. Such a rotation moves the wingtips (and everything else) in purely horizontal directions. As long as the airplane's center-of-mass motion is also horizontal, the rotation can only change the speeds, not the angles, of the airflow.

Now, things get more interesting when the direction of flight is not horizontal. Therefore let us consider a new example in which you are climbing while turning. That means your flight path is inclined above the horizontal. As before, you are turning to the left at a steady rate.

In any halfway-reasonable situation, the direction of flight will very nearly lie in the plane of yaw-wise rotations. Having it not exactly in the plane is just a distraction from the present topic, so I hereby define a new plane of "yaw-like" rotations which is defined by the direction of flight and the good old $Y$ axis (the wingtip-to-wingtip direction). The pitch-wise rotations remain the same, and we define a new plane of "roll-like" rotations perpendicular to the other two. We assume zero slip angle for simplicity.

As the airplane flies from point to point along its curving path, its heading must change. This is a rotatation in a purely horizontal plane. In climbing flight, the yaw-like direction is not exactly horizontal, so the turn is not pure yaw. The turn moves the inside wingtip horizontally backwards, relative to where it would be if there were no heading change. In contrast, a pure yaw-like rotation would have moved the wing back and down. Therefore we need not just leftward yaw-like rotation but also some rightward roll-like rotation to keep the wingtip moving along the actual flight path.

This roll-like motion means that (other things being equal) the inside wingtip would fly at a lower angle of attack during a climbing turn. Less lift would be produced. You need to deflect the ailerons to the outside to compensate.

Note that I said less lift "would be" produced, not "is" produced. That's because l'm assuming you have compensated with the ailerons, so that both wings are producing the same amount of lift, as they should. Remember that this is a steady turn, so no force is required to maintain the steady roll rate. (Remember, according to Newton's laws, an unbalanced force would create an acceleration in the roll-wise direction, which is not what is happening here.) There are widespread misconceptions about this. Because of the roll-like motion, the air will arrive at the two wings from two different directions. You deflect the ailerons, not in order to create a wing-versus-wing difference in the magnitude of lift, but rather to avoid creating such a difference.

The best you can do is to keep the magnitude of the lift the same. The direction of the lift will be twisted, as discussed in section 8.8.4; see in particular figure 8.7. You will need to deflect the rudder to overcome the resulting yawing moment. This will be in the usual direction: right rudder in proportion to right aileron deflection, and left rudder in proportion to left aileron deflection.

In a climbing turn, the differential relative wind combines with the differential wingtip velocity to create a large overbanking tendency. In an ordinary descending turn, the relative wind effect tends to oppose the velocity effect. In a spin, the differential relative wind is a key ingredient, as discussed in section 18.6.1, including figure 18.6. Also, section 9.7 analyzes climbing and descending turns in slightly different words and gives a numerical example.

### 19.6.5 Yaw Does Not Commute with Bank

As stated above, a rotation in a purely horizontal plane is not a pure yaw when the aircraft is not in a level attitude. In the previous section we considered the consequences of a non-level pitch attitude, but the same logic applies to a non-level bank attitude. The latter case is in some sense more significant, since although not all turns involve a non-level pitch attitude, they almost always involve a bank.

You could perform the required rotation by rolling to a level attitude, performing a pure yaw, and then rolling back to the banked attitude. This is not equivalent to performing a pure yaw while maintaining constant bank. For modest bank angles, the constant-bank maneuver is mostly pure yaw, but involves some rotation in the pitch-wise direction as well. Because of this pitch-wise rotation, the relative wind hits the wing and the tail at slightly different angles. You will need to pull back on the yoke slightly to compensate. This pull is in addition to whatever pull you might use for controlling airspeed during the turn. You can see that the two phenomena are definitely distinct, by the following argument: suppose that you maintain constant angle of attack during the turn, so that the required load factor is produced by increased airspeed not increased angle of attack. You would still need to pull back a little bit, to overcome the noncommutativity.

### 19.7 Torque and Moment

Just as Newton's first law says that to start an object moving you have to apply a force, there is a corresponding law that says to start an object turning you need to apply a torque.

You may have heard of the word "torque" in conjunction with left-turning tendency on takeoff, and you may have heard of the word "moment" in conjunction with weight \& balance problems. When pilots talk about moment, they usually mean a particular type of moment that is equal to a torque. In other contexts, there exist other types of moments that are not equal to torque; examples include moment of inertia and dipole moment. We don't need to discuss such things in detail, but you should be aware that they exist. In the present context, you can more-or-less assume that moment means torque. In particular,

- A rolling moment is a torque in the roll-wise direction.
- A pitching moment is a torque in the pitch-wise direction.
- A yawing moment is a torque in the yaw-wise direction.

A familiar example: fuel and cargo cause a pitching moment, depending on how far forward or aft they are loaded. By the same token, they will cause a rolling moment if they are loaded asymmetrically left or right.

Another familiar example: gyroscopic effects are known for causing yaw-wise torques. By the same token, they can cause pitch-wise torques as well.

Torque is not the same as force. Of the two, force is the more familiar concept. If you attach a string to an object and pull, the object is subjected to a force in the direction of the string. Force is measured in pounds or newtons.

To apply a torque, you need a force and a lever-arm. The amount of torque is defined by the following formula:

$$
\text { torque }=\text { arm } \square \text { force }
$$

where the arm (also called lever arm) is a vector representing the separation between the pivot-point ${ }^{-6}$ and the point where the force is applied. In this formula, we are multiplying vectors using the geometric wedge product..$\frac{7}{}$ The product is called a bivector, and is represented by an area, namely the area of the parallelogram spanned by the two vectors, as shown in figure 19.9. All five bivectors in the figure are equivalent, as you can confirm by counting squares.


Figure 19.9: Torque: Equivalent Bivectors

A vector (such as force) has geometric extent in one dimension. The drawing of a vector has a certain length. This is in contrast to scalars, which have no geometric extent. They are zerodimensional, and are drawn as points with no size.

A vector points in a definite direction. It is drawn with an arrowhead on one end.

A bivector (such as torque) has geometric extent in two dimensions. The drawing of a bivector has a certain area. In particular, the torque in figure 19.11 is represented by an area in the plane of the paper.

A bivector has a definite direction of circulation. It is drawn with arrowheads on its edges.

When constructing a bivector from two vectors, such as $A \square F$, you determine the direction of circulation by going in the $A$ direction then going in the $F$ direction, not vice versa. In particular, $F \square A$ $=-A \square F$, which tells us the two bivectors are equal-and-opposite.

When the force and the lever-arm are perpendicular, the magnitude of the torque is equal to the magnitude of the force times the length of the lever-arm, which makes things simple. If the two vectors are not perpendicular, pick one of them (say the force). Then keep the component of that
vector perpendicular to the other vector, throwing away the non-perpendicular component. What remains is two perpendicular vectors, and you can just multiply their magnitudes.

Torque is measured not in pounds but in footpounds (that is, feet times pounds); the corresponding metric unit is newtonmeters. ${ }^{8}$

Figure 19.10 shows a situation where all the forces and torques are in balance. On the right side of the bar, a group of three springs is exerting a force of 30 pounds. On the left side of the bar, there is a group of two springs (exerting a force of 20 pounds) and a single spring (exerting a force of 10 pounds). Since the total leftward force equals the total rightward force, the forces are in balance.


Figure 19.10: Forces and Torques in Balance
To show that the torques are in balance requires a separate check. Let's choose the point marked " $x$ " as our pivot point. The rightward force produces no torque, because it is attached right at the pivot point --- it has a zero-length lever arm. The group of two springs produces a counterclockwise torque, and the single spring produces a clockwise torque of the same magnitude, because even though it has half as much force it has twice the lever arm. The torques cancel. The system is in equilibrium.


## Figure 19.11: Forces in Balance but Torques NOT in Balance

Figure 19.11 shows a different situation. The forces are in balance ( 20 pounds to the right, 20 pounds total to the left) but the torques are not in balance. One of the left-pulling springs has twice the lever arm, producing a net clockwise torque. If you tried to set up a system like this, the bar would immediately start turning clockwise. The system is out of equilibrium.

### 19.8 Angular Momentum

The notion of angular momentum is the key to really understanding rotating objects.
Angular momentum is related to ordinary straight-line momentum in the same way that torque is related to ordinary straight-line force. Here is a summary of the correspondences:

## Straight-line concept

## Force

Momentum

The ordinary momentum of a system won't change unless a force is applied.

Force equals momentum per unit time.

## Angular concept

Torque (equals force times lever arm)
Angular momentum (equals ordinary momentum times lever arm)

The angular momentum of a system won't change unless a torque is applied.

Torque equals angular momentum per unit time.

When I give lectures, I illustrate conservation of angular momentum using a demo you can easily set up for yourself. As illustrated in figure 19.12, tie some kite string to a small bean-bag and swing it in a circle. When you pull on the free end of the string (reducing the radius of the circle) the bean-bag speeds up. When you let out the string (increasing the radius of the circle) the bean-bag slows down. ${ }^{-}$


Figure 19.12: Conservation of Angular Momentum
In typical textbooks, conservation of angular momentum is exemplified by spinning ice skaters, but I find it easier to travel with a bean-bag (rather than an ice skater) in my luggage.

In the demonstration, there are some minor torques due to friction than will eventually slow down the bean-bag whether or not you shorten or lengthen the string, but if you perform the experiment quickly enough the torques can be neglected, and the angular momentum of the system is more or less constant. Therefore, if you decrease the lever arm by a factor of $N$, the straight-line momentum must increase by a factor of $N$ (since their product cannot change). ${ }^{10}$

Since the tangential velocity increases by a factor of $N$, and the radius decreases by a factor of $N$, the rate of turn (degrees per second) increases by a factor of $N$ squared.

The energy of the system also increases by a factor of $N$ squared. You can feel that you added energy to the system when you pull on the string, pulling against tension.

So far we have analyzed the situation from the point of view of a bystander in a non-rotating reference frame. You can reach the same conclusion by analyzing the situation in the rotating reference frame, as would apply to an ant riding on the bean-bag. The ant would say that as the string is pulled in, the bean-bag accelerates sideways because of the Coriolis effect, as discussed in section 19.3.

Conservation of angular momentum applies to airplanes as well as bean-bags. For instance, consider an airplane in a flat spin, as discussed in section 18.6.4. In order to recover from the spin, you need to push the nose down. This means whatever mass is in the nose and tail will move closer to the axis of rotation. The angular momentum of the airplane doesn't change (in the short run), so the rotation will speed up (in the short run). More rotation may seem like the opposite of what you wanted, but remember you are trying to get rid of angular momentum, not just angular rate. You should persevere and force the nose down. Then the aerodynamic forces (or, rather, torques) will carry angular momentum out of the system and the rotation will decrease.

Angular momentum is a bivector, like torque (section 19.7). It lies more-or-less ${ }^{11}$ in the plane of rotation.

### 19.9 Gyroscopes

### 19.9.1 Precession

For any normal object (such as a book) if you apply a force in a given direction, it will respond with motion in that direction. People are so accustomed to this behavior that they lose sight of the fact that force and motion are not exactly the same thing, and they don't always go together.

In particular, for a gyroscope, if you apply a torque in one direction it will respond with motion in a different direction. When I give my "See How It Flies" lectures, I carry around a bicycle wheel with handles, as shown in figure 19.13. The indicated direction of spin corresponds to a normal American engine and propeller, if the nose of the airplane is toward the left side of the diagram.


Figure 19.13: Bicycle Wheel with Handles
To demonstrate the remarkable behavior of a gyroscope, I stand behind the "propeller" (on the right side of the diagram) and support its weight by lifting the rear handle only. The force of gravity acts on the center of the system, so there is a pure nose-down / tail-up pitching moment. If this were a normal, non-spinning object, it would respond by pitching in the obvious way, but the gyroscope actually responds with a pure yawing motion. I have to turn around and around to my left to stay behind the wheel.

It is really quite amazing that the wheel does not pitch down. Even though I am applying a pitch-wise torque, the wheel doesn't pitch down; it just yaws around and around.


Figure 19.14: Gyroscopic Precession
This phenomenon, where a gyro responds to a torque in one direction with a motion in another direction, is called gyroscopic precession.

For a gyroscope, a torque in the pitch-wise direction produces a motion in the yaw-wise direction. If you try to raise the tail of a real airplane using flippers alone, it will yaw to the left because of precession.

This effect is particularly noticeable early in the takeoff roll in a taildragger, when you raise the tail to keep the airplane on the ground while you build up speed. If the airplane were an ordinary nonspinning object, you could raise the tail just by pushing on the yoke. But note that airflow over the flippers does not actually dictate the motion of the airplane; it just produces a torque in the pitch-wise direction. When you combine this torque to the angular momentum of the engine, the result is pronounced precession to the left. You need to apply right rudder to compensate.

Another place where this is noticeable is during power-on stall demonstrations. You need a downward pitch-wise torque to make the non-rotating parts of the airplane pitch down. But this same pitch-wise torque, when added to the angular momentum of the engine, causes yaw-wise precession to the left. You need right rudder to compensate.

To get a gyroscope to actually move in the pitch-wise direction, you need to apply a torque in the yaw-wise direction --- using the rudder.

Of course, an airplane has some ordinary non-rotating mass in addition to its gyroscopic properties. In order to lift this ordinary mass you need to use the flippers. Therefore, the tail-raising maneuver
requires both flippers and rudder --- flippers to change the pitch of the ordinary mass, and rudder to change the pitch of the gyroscope.

### 19.9.2 Precession: Which Way and How Much

Let's try to understand what causes precession, so we can predict which way the airplane will precess, and how much. Consider what happens when a torque is applied for a certain small time interval (one second or so). This will contribute some angular momentum to the system. Remember: torque is angular momentum per unit time. Then we just add this contribution to the initial angular momentum, and the result is the final angular momentum.

Angular momentum is a bivector. Figure 19.15 shows the bivectors involved in the precession, and figure 19.16 is an exploded view showing how to add bivectors. We put them edge-to-edge, in analogy to the way we add ordinary vectors by placing them tip-to-tail. In this example, edge $b$ adds tip-to-tail to edge $x$ to form the top edge of the sum. Similarly, edge $z$ adds to edge $c$ to form the bottom edge of the sum. Edge $c$ cancels edge $w$ since they are equal and opposite. Edges $a$ and $y$ survive unchanged to become the vertical edges of the sum.


Figure 19.15: Angular Momentum Explains Precession


## Figure 19.16: Addition of Bivectors -- Exploded View

We see that the new angular momentum differs from the old angular momentum by a yaw to the left. That's the correct answer.

During subsequent time intervals, the torque will be a new direction because the whole system has rotated. The successive changes will cause the system (wheel, axle, and everything attached to it) to keep turning in the horizontal plane, yawing to the left.

Beware: This gyroscope law might seem roughly similar to the Coriolis effect (force in one direction, motion in a perpendicular direction) but they do not represent the same physics. The Coriolis law only applies to objects that are moving relative to a rotating observer. In contrast, the gyroscope law applies to a stationary observer, and a wheel precesses even though no part of the wheel is moving relative to other parts.

Gyroscopic effects only occur when the there is a change in the orientation of the gyro's plane of rotation. You can take a gyro and transport it north/south, east/west, or up/down, without causing any precession, as long as the gyro's plane of rotation remains parallel to the original plane of rotation. You can even roll an airplane without seeing gyroscopic effects due to engine rotation, since the roll
leaves the engine's plane of rotation undisturbed.
You can figure it out by adding the bivectors. Right rudder deflection will cause a pitch-wise precession in the nose-down / tail-up direction. Pushing on the yoke causes a yaw-wise precession to the left.

If you have a lightweight airframe and a heavy, rapidly spinning propeller, watch out: the flippers will cause yawing motion and the rudder will cause pitching motion.

If you want to make a gyro change orientation quickly, it will take more torque than doing it slowly.

### 19.9.3 Inertial Platform

We now consider what happens when a gyro is not subjected to any large torques.
Suppose we support a gyroscope on gimbals. The gimbals support its weight but do not transmit any torques to it, even if the airplane to which the gimbals are mounted is turning. We call this a free gyro since it is free to not turn when the airplane turns.

Even though the gyro is small, it has a huge amount of angular momentum, because it is spinning so rapidly. Any small torque applied to the gyro (because of inevitable imperfections in the gimbals) will, over time, change the angular momentum --- but over reasonably short times the change is negligible compared to the total.

In such a situation, the gyro will tend to maintain fixed orientation in space. We say that the gyro is an inertial platform with respect to rotations. ${ }^{12}$ Other books say the gyro exhibits rigidity in space but that expression seems a bit odd to me.

### 19.10 Gyroscopic Instruments

We now discuss the principles of operation of the three main gyroscopic instruments: artificial horizon (attitude indicator), directional gyro (heading indicator), and rate of turn gyro (turn needle or turn coordinator).

### 19.10.1 Heading Indicator

The directional gyro is a free gyro. It establishes an inertial platform.
The gyro spins in some vertical plane; that is, its angular momentum vector points in some arbitrary horizontal direction. A system of gears measures the angle that the angular momentum vector makes in the $X Y$ plane ${ }^{\frac{13}{3}}$ and displays it to the pilot. The trick is to measure the angle and support the gyro while minimizing the accidental torques on it. Imperfections in the mechanism cause the gyro to precess; therefore, every so often the heading indication must be corrected, typically by reference to a magnetic compass.

### 19.10.2 Artificial Horizon

The artificial horizon (also known as the attitude indicator) is another free gyro. This gyro's plane of rotation is horizontal; that is, its angular momentum vector is vertical. A mechanical linkage measures the angle that this vector makes in the $Y Z$ (bank) and $X Z$ (pitch) planes, and displays it to the pilot.

It is instructive to compare the horizon gyro (which tells you which way is "down") with the inclinometer ball or a plumb-bob on a string (which has a different notion of which way is "down"). The distinction is that the plumb-bob tells you which way is E-down, while the gyro is designed to tell you which way is N -down (toward the center of the earth). Whenever the airplane is being accelerated (e.g. during the takeoff roll or during a turn), the two directions are quite different. As seen in figure 19.17, during a turn the E-down vector gets centrifuged to the outside of the turn; the N-down vector always points to the center of the earth.


Figure 19.17: E-Down versus N-down During a Turn
As you can see in figure 19.17,

- sometimes E-down points a little to the north of N -down,
- sometimes E-down points a little to the west of N-down,
- sometimes E-down points a little to the south of N -down,
- sometimes E-down points a little to the east of N -down.

To a first approximation, the horizon gyro works just by remembering which way is N-down. However, no gyro can remember anything forever, so the instrument contains an "erecting mechanism" that makes continual small adjustments. You would like it to align the gyro axis with N -down --- but the mechanism doesn't know which way is N-down! It knows which way is E-down (the same way the plumb-bob does), but according to Einstein's principle of equivalence, it cannot possibly know what components of E-down are due to gravity and what components are due to acceleration. The erecting mechanism does, in fact, continually nudge the gyro axis toward E -down, but the result is a good approximation to N -down, for the following reason: if you average the E-down vectors over an entire turn, they average out to N -down.

If you average the discrepancies over an entire turn, they cancel. This is why a gyro is vastly more valuable than a plumb-bob: The gyro can perform long-term averaging, whereas a plumb-bob can't.

## * Artificial Horizon Errors

Of course, if you only make half a turn, the discrepancies don't average to zero, and the attitude indicator will be slightly inaccurate for a while. Analogous errors occur during takeoff, because the gyro's estimate of "down" gets dragged backwards by the acceleration, so the artificial horizon will be a little bit below the true forward horizon for a while thereafter. The averaging time for a typical instrument is about five minutes.

Sometimes you find an old, worn-out instrument in which the gyro isn't spinning as fast as it should. As a result, its memory gets shorter, and the systematic errors become larger.

### 19.10.3 Rate-of-Turn Gyro

There are two slightly different types of rate-of-turn gyro: (a) the rate-of-turn needle, and (b) the turn coordinator.

In both cases, the gyro is not free; it is a rate gyro. That is, its plane of rotation is more-or-less firmly attached to the airplane. It does not have gimbals. It is forced to change orientation when the airplane yaws. ${ }^{14}$ The instrument measures how much torque is required to re-orient the gyro.

Sometimes the rate-of-turn needle is built to spin in the pitch-wise ( $Z X$ ) plane, in which case the airplane's yawing motion requires a torque in the roll-wise ( $Y Z$ ) direction. Other models spin in the roll-wise $(Y Z)$ plane, in which case yaw requires a torque in the pitch-wise $(Z X)$ direction. In principle, the spin and the torque could be in any pair of planes perpendicular each other and perpendicular to the yaw-wise ( $X Y$ ) plane. ${ }^{15}$

The required torque is proportional to (a) the rate of change of orientation, and (b) the angular momentum of the gyro. Therefore an accurate rate-of-turn gyro must spin at exactly the right speed, not too fast or too slow. (This is in contrast to the directional gyro and the artificial horizon gyro, which just have to spin "fast enough".)

Many rate gyros incorporate a sneaky trick. They spin around the pitch-wise $(Z X)$ plane, with the top of the gyro spinning toward the rear. They also use a spring that is weak enough to allow the gyro to precess a little in the roll-wise ( $Y Z$ ) direction. In a turn to the left, precession will tilt the gyro a little to the right. That means that during a turn, the gyro's tilt compensates for the airplane's bank, leaving the gyro somewhat more aligned with the earth's vertical axis. The goal, apparently, is to create an instrument that more nearly indicates heading change (relative to the earth's vertical axis) rather than simply rotation in the airplane's yaw-wise ( $X Y$ ) plane, which is not exactly horizontal during the turn. Since the relationship between bank angle and rate of turn depends on airspeed, load factor, et cetera, this trick can't possibly achieve the goal except under special conditions.

The turn coordinator is very similar to the rate-of-turn needle. It displays a miniature airplane instead of a needle. The key operational difference is that it is slightly sensitive to rate of roll as well as rate of heading change. To create such an instrument, all you have to do is take a rate-of-turn instrument, tilt the mechanism nose-up by 20 or 30 degrees, and change the display.

The advantage of a turn coordinator is that it helps you anticipate what actions you need to take. That is, if the airplane has its wings level but is rolling to the right, it will probably be turning to the right pretty soon, so you might want to apply some aileron deflection. The disadvantage has to do with turbulence. Choppy air oftentimes causes the airplane to roll continually left and right. The roll rate can be significant, even if the bank angle never gets very large. The chop has relatively little effect on the heading. In such conditions a plain old rate-of-turn needle gives a more stable indication than a turn coordinator does.

It is rather unfortunate that the display on a turn coordinator is a miniature airplane that banks left and right. This leads some people to assume, incorrectly, that the instrument indicates bank angle, which it most definitely does not, as you can demonstrate by performing a boat turn (section 8.10).

This is traditionally expressed in terms of an "action" and an "equal and opposite reaction", but the meaning of those words has drifted over the centuries. Momentum is the modern term.

Troublemakers sometimes point out that lift actually is slightly reduced in a steady descent, since part of the weight is being supported by drag. To this I retort: (a) this is an obscure technicality, based on details of the definitions of the four forces (as given in section 4.1); (b) the magnitude of the reduction is negligible in ordinary flying, (c) the lift is reduced for climbs as well as descents --- so this technicality certainly does not explain the motion, and (d) when we consider the total upward force, there is no reduction.

It is easy to find hand-waving explanations of the Coriolis effect that overlook one or the other of the two contributions, and are therefore off by a factor of two. Beware.

If you consider multiple widely-separated points, you can distinguish gravity versus centrifugity versus straight-line acceleration by checking for nonuniformities in the fields. However, an airplane is so small compared to the planet, and so small compared to its turning radius, that these nonuniformities do not provide a very practical way of telling one field from another.

It might be relevant if you are designing an airplane or a flight simulator.
The pivot-point is also known as the datum. In ordinary cases (specifically, when you know the forces are in balance and you are trying to figure out whether the torques are in balance) it doesn't matter what point in the airplane you choose as the pivot-point, provided you measure all lever arms from the same point.

Some other books try to calculate the torque using a "cross product" but the wedge product is much nicer. The wedge product is in some sense complimentary to the dot product used in section 4.5.

Sometimes you see these written as hyphenated words (foot-pounds or newton-meters) in which case the hyphen should not be mistaken for a minus sign. A foot-pound is a foot times a pound, not a foot minus a pound.

It is best to feed the string through a small smooth tube, rather than just your bare hand. You might use a poultry baster, or the axial hole in a spool of thread.

The bean-bag acquires the necessary straight-line momentum, and energy, via the string. It cannot acquire angular momentum from the string, since that would require a lever arm perpendicular to the force. Since the string can only exert a force parallel to itself, the lever arm is zero, so the torque is zero.

For an object rotating around an axis of symmetry, the angular momentum lies exactly in the plane of rotation; for odd off-axis rotations this might not be true.

An even fancier inertial platform would keep a position (not just orientation) independent of straight-line accelerations.

See figure 19.8 for the definition of the $X, Y$, and $Z$ directions.
The instrument is not directly sensitive to any change in the direction the airplane is going, just to changes in the direction it is pointing.

The $X, Y$, and $Z$ directions are defined in figure 19.8.

## 20 The Atmosphere

If you don't like the weather in Ithaca, just wait a few minutes. It'll get worse.
--- apologies to Mark Twain

### 20.1 Circulation Around Fronts and Low Pressure Centers

Because the earth is spinning and the air is moving, there are significant Coriolis effects. ${ }^{1}$ You'll never understand how weather systems work unless you pay attention to this.

Based on their everyday indoor experience, people think they understand how air behaves:

- They know that the stream of air from a fan moves in a straight line, with no particular tendency to curve right or left.
- They know that once the fan is switched off, the airflow won't last very long or travel very far before being overcome by friction.

However, when we consider the outdoor airflow patterns that Mother Nature creates, the story changes completely. In a chunk of air that is many miles across, a mile thick, and a mile away from the surface, there can be airflow patterns that last for hours or days, because there is so much more inertia and so much less friction. During these hours or days, the earth will rotate quite a bit, so Coriolis effects will be very important.

We are accustomed to seeing the rotation of storm systems depicted on the evening news, but you should remember that even a chunk of air that appears absolutely still on the weather map is rotating, because of the rotation of the earth as a whole. Any chunk of air that appears to rotate on the map must be rotating faster or slower than the underlying surface. (In particular, the air in a storm generally rotates faster, not slower.)

Note: In this chapter, I will use the § symbol to indicate words that are correct in the northern hemisphere but which need to be reversed in the southern hemisphere. Readers in the northern hemisphere can ignore the § symbol.

### 20.1.1 Flow Around a Low

Suppose we start out in a situation where there is no wind, and where everything is in equilibrium. We choose the rotating Earth as our reference frame, which is a traditional and sensible choice. In this rotating frame we observe a centrifugal field, as well as the usual gravitational field, but the air has long ago distributed itself so that its pressure is in equilibrium with those fields.

Then suppose the pressure is suddenly changed, so there is a region where the pressure is lower than the aforementioned equilibrium pressure.

In some cases the low pressure region is roughly the same size in every direction, in which case it is called a low pressure center (or simply a low) and is marked with a big "L" on weather maps. In other cases, the low pressure region is quite long and skinny, in which case it is called a trough and is marked "trof" on the maps. See figure 20.1.


Figure 20.1: Initial Force near a Low Pressure Region
In either case, we have a pressure gradient.. Each air parcel is subjected to an unbalanced force due to the pressure gradient.

Initially, each air parcel moves directly inward, in the direction of the pressure gradient, but whenever it moves it is subject to large sideways Coriolis forces, as shown in figure figure 20.2. Before long, the motion is almost pure counterclockwise§ circulation around the low, as shown in figure 20.3, and this pattern persists throughout most of the life of the low-pressure region. If you face downwind at locations such as the one marked $A$, the pressure gradient toward the left§ is just balanced by the Coriolis force to the right§, and the wind blows in a straight line parallel to the trough. At locations such as the one marked $B$, the pressure gradient is stronger than the Coriolis force. The net force deflects the air.


Figure 20.2: Initial Motion near a Low Pressure Region


Figure 20.3: Steady Motion near a Low Pressure Region
Now we must must account for friction (in addition to the other forces just mentioned). The direction of the frictional force will be opposite to the direction of motion. This will reduce the circulatory velocity. This allows the air to gradually spiral inward.

The unsophisticated idea that air should flow from a high pressure region toward a low pressure region is only correct in the very lowest layers of the atmosphere, where friction is dominant. If it weren't for friction, the low would never get filled in. At any reasonable altitude, friction is negligible --so the air aloft just spins around and around the low pressure region.

The astute reader may have noticed a similarity between the air in figure 20.2 and the bean-bag in figure 19.12. In one case, something gets pulled inwards and increases its circulatory motion "because" of Coriolis force, and in the other case something gets pulled inwards and increases its circulatory motion "because" of conservation of angular momentum. For a bean-bag, you can analyze it either way, and get the same answer. Also for a simple low-pressure center, you can analyze it either way, and get the same answer. For a trough, however, there is no convenient way to apply the conservation argument.

In any case, please do not get the idea that the air spins around a low partly because of conservation of angular momentum and partly because of the Coriolis force. Those are just two ways of looking at the same thing; they are not cumulative.

### 20.1.2 Fronts and Troughs

As mentioned above, whenever the wind is blowing in a more-or-less straight line, there must low pressure on the left§ to balance the Coriolis force to the right§ (assuming you are facing downwind). In particular, the classic cold front wind pattern (shown in figure 20.4) is associated with a trough, (as shown in figure 20.5). The force generated by the low pressure is the only thing that could set up the characteristic frontal flow pattern.


Figure 20.4: Wind Near a Front


Figure 20.5: Pressure Near a Front


Figure 20.6: Temperature Near a Front
The wind shift is what defines the existence of the front. Air flows one way on one side of the front, and the other way on the other side (as shown in figure 20.4).

Usually the front is oriented approximately north/south, and the whole system is being carried west-to-east by the prevailing westerlies. In this case, we have the classic cold front scenario, as shown in figure 20.4, figure 20.5, and figure 20.6. Ahead of the front, warm moist air flows in from the south§. Behind the front, the cold dry air flows in from the north§. Therefore the temperature drops when the front passes. In between cold fronts, there is typically a non-frontal gradual warming trend, with light winds.

You can use wind patterns to your advantage when you fly cross-country. If there is a front or a pressure center near your route, explore the winds aloft forecasts. Start by choosing a route that keeps the low pressure to your left§. By adjusting your altitude and/or route you can often find a substantial tailwind (or at least a substantially decreased headwind).

Note: by ancient tradition, meteorologists name winds by the direction from whence they come. A south wind (or southerly wind) blows from south to north. Almost everything else is named the other way. An aircraft on a southerly heading is flying toward the south. Physicists and mathematicians name all vectors by the direction toward which they point. To avoid confusion, it is better to say "wind
from the south" rather than "south wind".
A warm front is in many ways the same as a cold front. It is certainly not the opposite of a cold front. In particular, it is also a trough, and has the same cyclonic flow pattern.

A warm front typically results when a piece of normal cold front gets caught and spun backwards by the east-to-west flow just north§ of a strong low pressure center, as shown in figure 20.7. That is, near the low pressure center, the wind circulating around the center is stronger than the overall west-to-east drift of the whole system.


Figure 20.7: Warm Front
If a warm front passes a given point, a cold front must have passed through a day or so earlier. The converse does not hold --- cold front passage does not mean you should expect a warm front a day or so later. More commonly, the pressure is more-or-less equally low along most of the trough. There will be no warm front, and the cold front will be followed by fair weather until the next cold front.

Low pressure --- including cold fronts and warm fronts --- is associated with bad weather for a simple reason. The low pressure was created by an updraft that removed some of the air, carrying it up to the stratosphere. The air cools adiabatically as it rises. When it cools to its dew point, clouds and precipitation result. The latent heat of condensation makes the air warmer than its surroundings, strengthening the updraft.

Ascending air $\square$ low pressure at the surface Ascending air $\square$ clouds

The return flow down from the stratosphere (high pressure, very dry descending air, and no clouds) generally occurs over a wide area, not concentrated into any sort of front. There is no sudden wind shift, and no sudden change in temperature. This is not considered "significant weather" and is not marked on the charts at all.

### 20.2 Pressure and Winds Aloft

Air shrinks when it gets cold. This simple idea has some important consequences. It affects your altimeter, as will be discussed in section 20.2.4. It also explains some basic facts about the winds aloft, which we will discuss now.

### 20.2.1 Thermal Gradient Wind

Most non-pilots are not very aware of the winds aloft. Any pilot who has every flown westbound in the winter is keenly aware of some basic facts:

- The winds aloft tend to come from the west.
- They are much stronger in the winter.
- They get stronger and stronger as altitude increases.

A typical situation is shown in figure 20.8. In January, the average temperature in Vero Beach, Florida, is about 15 Centigrade (59 Fahrenheit), while the average temperature in Oshkosh, Wisconsin is about minus 10 Centigrade (14 Fahrenheit). Imagine a day where surface winds are very weak, and the sea-level barometric pressure is the same everywhere, namely 1013 millibars (29.92 inches of mercury).


Figure 20.8: Thermal Gradient Wind
The pressure above Vero Beach will decrease with altitude. According to the International Standard Atmosphere (ISA), we expect the pressure to be 697 millibars at 10,000 feet.

Of course the pressure above Oshkosh will decrease with altitude, too, but it will not exactly follow the ISA, because the air is 25 centigrade colder than standard. Air shrinks when it gets cold. In the figure, I have drawn a stack of ten boxes at each site. Each box at VRB contains the same number of air molecules as the corresponding box at OSH. ${ }^{-3}$ The pile of boxes is shorter at OSH than it is at VRB.

The fact that the OSH air column has shrunk (while the VRB air column has not) produces a big effect on the winds aloft. As we mentioned above, the pressure at VRB is 697 millibars at 10,000 feet. In contrast, the pressure at OSH is 672 millibars at the same altitude --- a difference of 25 millibars.

This puts a huge force on the air. This force produces a motion, namely a wind of 28 knots out of the west. (Once again, the Coriolis effect is at work: during most of the life of this pressure pattern, the wind flows from west to east, producing a Coriolis force toward the south, which just balances the pressure-gradient force toward the north.) This is the average wind at 10,000 feet, everywhere between VRB and OSH.

More generally, suppose surface pressures are reasonably uniform (which usually the case) and temperatures are not uniform (which is usually the case, especially in winter). If you have low temperature on your left§ and high temperature on your right§, you will have a tailwind aloft. The higher you go, the stronger the wind. This is called thermal gradient wind.

The wind speed will be proportional to the temperature gradient. Above a large airmass with uniform temperature, there will be no thermal gradient wind. But if there is a front between a warm airmass and a cold airmass, there will be a large temperature change over a short distance, and this can lead to truly enormous winds aloft.

In July, OSH warms up considerably, to about 20 centigrade, while VRB only warms up slightly, to about 25 centigrade. This is why the thermal gradient winds are typically much weaker in summer than in winter --- only about 5 knots on the average at 10,000 feet.

In reality, the temperature change from Florida to Wisconsin does not occur perfectly smoothly; there may be large regions of relatively uniform temperature separated by rather abrupt temperature gradients --- cold fronts or warm fronts. Above the uniform regions the thermal gradient winds will be weak, while above the fronts they will be much stronger.

For simplicity, the foregoing discussion assumed the sea-level pressure was the same everywhere. It also assumed that the temperature profile above any given point was determined by the surface temperature and the "standard atmosphere" lapse rate. You don't need to worry about such details; as a pilot you don't need to calculate your own winds-aloft forecasts. The purpose here is to make the official forecasts less surprising, less confusing, and easier to remember.

### 20.2.2 Altimetry

Several different notions of "altitude" are used in aviation.
We start with true altitude, which is the simplest. This is what non-pilots think of as "the" altitude or elevation, namely height above sea level, as measured with an accurate ruler. True altitude is labelled MSL (referring to Mean Sea Level). For instance, when they say that the elevation of Aspen is 7820 feet MSL, that is a true altitude.

Before proceeding, we need to introduce the notion of international standard atmosphere (ISA). The ISA is a set of formulas that define a certain temperature and pressure as a function of altitude. For example, at zero altitude, the ISA temperature is 15 degrees centigrade, and the ISA pressure is 1013.25 millibars, or equivalently 29.92126 inches of mercury. As the altitude increases, the ISA temperature decreases at a rate of 6.5 degrees centigrade per kilometer, or very nearly 2 degrees $C$ per thousand feet. The pressure at 18,000 feet is very nearly half of the sea-level pressure, and the pressure at 36,000 feet is somewhat less than one quarter of the sea-level pressure -- so you can see the pressure is falling off slightly faster than exponentially. If you want additional details on this, a good place to look is the Aviation Formulary web site.

Remember, the ISA is an imaginary, mathematical construction. However, the formulas were chosen so that the ISA is fairly close to the average properties of the real atmosphere.

Now we can define the notion of pressure altitude. This is not really an altitude; it is just a way of describing pressure. Specifically, you measure the pressure, and then figure out how high you would have to go in the international standard atmosphere to find that pressure. That height is called the pressure altitude. One tricky thing is that low pressure corresponds to high pressure altitude and vice versa.

Pressure altitude (i.e. pressure) is worth knowing for several reasons. For one thing, if the pressure altitude is too high, you will have trouble breathing. The regulations on oxygen usage are expressed in terms of pressure altitude. Also, engine performance is sensitive to pressure altitude (among other factors). Thirdly, at high altitudes, pressure altitude is used for vertical separation of air traffic. This works fine, even though the pressure altitude may be significantly different from the true altitude (because on any given day, the actual atmosphere may be different from the ISA). The point is that two aircraft at the same pressure level will be at the same altitude, and two aircraft with "enough" difference in pressure altitude will have "enough" difference in true altitude.

To determine your pressure altitude, set the Kollsman window on your altimeter to the standard value: 29.92 inches, or equivalently 1013 millibars. Then the reading on the instrument will be the pressure altitude (plus or minus nonidealities, as discussed in section 20.2.3).

This brings us to the subject of calibrated altitude and indicated altitude. At low altitudes -- when we need to worry about obstacle clearance, not just traffic separation -- pressure altitude is not good enough, because the pressure at any given true altitude varies with the weather. The solution is to
use indicated altitude, which is based on pressure (which is convenient to measure), but with most of the weather-dependence factored out. To determine your indicated altitude, obtain a so-called altimeter setting from an appropriate nearby weather-reporting station, and dial it into the Kollsman window on your altimeter. Then the reading on the instrument will be the indicated altitude. (Calibrated altitude is the same thing, but does not include nonidealities, whereas indicated altitude is disturbed by nonidealities of the sort discussed in section 20.2.3.)

The altimeter setting is arranged so that right at the reporting station, calibrated altitude agrees exactly with the station elevation. By extension, if you are reasonably close to the station, your calibrated altitude should be a reasonable estimate of your true altitude ... although not necessarily good enough, as discussed in section 20.2.3 and section 20.2.4).

Next we turn to the notion of absolute altitude. This is defined to be the height above the surface of the earth. Here is a useful mnemonic for keeping the names straight: the Absolute Altitude is what you see on the rAdAr altimeter. Absolute altitude is labelled "AGL" (above ground level). It is much less useful than you might have guessed. One major problem is that there may be trees and structures that stick up above the surface of the earth, and absolute altitude does not account for them. Another problem is that the surface of the earth is uneven, and if you tried to maintain a constant absolute altitude, it might require wild changes in your true altitude, which would play havoc with your energy budget. Therefore the usual practice in general aviation is to figure out a suitable indicated altitude and stick to it.

Another type of altitude is altitude above field elevation, where field means airfield, i.e. airport. This is similar to absolute altitude, but much more widely used. For instance, the traffic-pattern altitude might be specified as 1000 feet above field elevation. Also, weather reports give the ceiling in terms of height above field elevation. This is definitely not the same as absolute altitude, because if there are hills near the field, 1000 feet above the field might be zero feet above the terrain. Altitude above field elevation should be labelled "AFE" but much more commonly it is labelled "AGL". If the terrain is hilly "AGL" is a serious misnomer.

Finally we come to the notion of density altitude. This is not really an altitude; it is just a way of describing density. The official definition works like this: you measure the density, and then figure out how high you would have to go in the ISA to find that density. That height is called the density altitude. Beware that low density corresponds to high density altitude and vice versa.

Operationally, you can get a decent estimate of the density altitude by measuring the pressure altitude and temperature, and then calculating the density altitude using the graphs or tables in your POH . This is only an estimate, because it doesn't account for humidity, but it is close enough for most purposes.

Density altitude is worth knowing for several reasons. For one thing, the TAS/CAS relationship is determined by density. Secondly, engine performance depends strongly on density (as well as pressure and other factors). Obviously TAS and engine performance are relevant to every phase of flight -- sometimes critically important.

### 20.2.3 Altimeter Errors

As discussed in the previous section, an aircraft altimeter does not measure true altitude. It really measures pressure, which is related to altitude, but it's not quite the same thing.

In order to estimate the true altitude, the altimeter depends two factors: the pressure, and the
altimeter setting in the Kollsman window. The altimeter setting is needed to correct for local variations in barometric pressure. You should set this on the runway before takeoff, and for extended flights you should get updated settings via radio. If you neglect this, you could find yourself at a too-low altitude, if you fly to a region where the barometric pressure is lower. The mnemonic is: "High to low, look out below".

Altimeters are not perfect. Even if the altimeter and airplane were inspected yesterday, and found to be within tolerances,

- The altimeter could be off by 30 feet when it reads 2500 (according to the tolerances in FAR 43 Appendix E).
- If the airplane is moving at 100 knots, the indicated altitude could be off by another 30 feet, due to nonidealities in the arrangement of the static port (FAR 23.1325).
- If the airplane is descending at 750 FPM, the altimeter could be off by an additional 70 feet, due to friction in the mechanism (FAR 43 Appendix E).
- There could be 30 feet of hysteresis, if you have recently descended from a very high altitude (FAR 43 Appendix E).
- Wind flowing over a nice airfoil-shaped hill can produce low pressure there. A 35-knot wind could produce a 50 -foot altimetry error. See section 3.4.1.

The first item could be off in either direction, but the other items will almost certainly be off in the bad direction when you are descending. Also, if the airplane has been in service for a few months since the last inspection, the calibration could have drifted a bit. All in all, it would be perfectly plausible to find that your altimeter was off by 50 feet when parked on the ground, and off by 200 feet in descending flight over hilly terrain.

### 20.2.4 High Altimeter due to Low Temperature

The altimeter measures a pressure and converts it to a so-called altitude. The conversion is based on the assumption that the actual atmospheric pressure varies with altitude the same way the the standard atmosphere would. The pressure decreases by roughly $3.5 \%$ per thousand feet, more or less, depending on temperature.

The problem is that the instrument does not account for nonstandard temperature. Therefore if you set the altimeter to indicate correctly on the runway at a cold place, it will be inaccurate in flight. It will indicate that you are higher than you really are. This could get you into trouble if you are relying on the altimeter for terrain clearance. The mnemonic is HALT --- High Altimeter because of Low Temperature.

As an example: Suppose you are flying an instrument approach into Saranac Lake, NY, according to the FAA-approved "Localizer Runway 23" procedure. The airport elevation is 1663 feet. You obtain an altimeter setting from the airport by radio, since you want your altimeter to be as accurate as possible when you reach the runway.

You also learn that the surface temperature is -32 Centigrade, which is rather cold but not unheard-of at this location. That means the atmosphere is about 45 C colder than the standard atmosphere. That in turn means the air has shrunk by about $16 \%$. Throughout the approach, you will be too low by an amount that is $16 \%$ of your height above the airport.

The procedure calls for crossing the outer marker at 3600 MSL and then descending to 2820 MSL, which is the Minimum Descent Altitude. That means that on final approach, you are supposed to be

1157 feet above the airport. If you blindly trust your altimeter, you will be 1157 "shrunken feet" above the airport, which is only about 980 real feet. You will be 180 real feet ( 210 shrunken feet) lower than you think. To put that number in perspective, remember that localizer approaches are designed to provide only 250 feet of obstacle clearance. ${ }^{4}$

You must combine this HALT error with the ordinary altimetry errors discussed in section 20.2.3. The combination means you could be 400 feet lower than what the altimeter indicates --- well below the protected airspace. You could hit the trees on Blue Hill, 3.9 nm northeast of the airport.

Indeed, you may be wondering why there haven't been lots of crashes already -- especially since the Minimum Descent Altitude used to be lower (1117 feet, until mid-year 2001). Possible explanations include:

- Most people use the ILS approach instead of the localizer approach. That provides electronic vertical guidance that isn't affected by temperature.
- In winter, the real atmosphere usually has a smaller lapse rate than the standard atmosphere, especially at the lower altitudes, so the errors are usually less than what simple theory would suggest.
- The FAA has overestimated the height of the trees. They routinely assume there will be small structures and trees rising 200 feet above the land surface, but the trees on Blue Hill are probably closer to 100 feet. This is helpful, but we shouldn't rely on it. The trees are still growing, and other trees in the vicinity are over 150 feet tall. Furthermore, if somebody built a 190-foot tower atop Blue Hill, the FAA would not change the Minimum Descent Altitude for this procedure, and there would be problems for sure.
- The new 1157 -foot Minimum Descent Altitude is about 40 feet higher than what you would expect just based on the height of the hill. I'm told this represents an allowance for the effect of wind blowing over hilly terrain as mentioned above.

Even if people don't "usually" crash, we still need to do something to increase the margin for error.
There is an obvious way to improve the situation: In cold weather, you need to apply temperature compensation to all critical obstacle-clearance altitudes.

You can do an approximate calculation in your head: If it's cold, add 10\%. If it's really, really cold, add $20 \%$. Approximate compensation is a whole lot better than no compensation.

The percentages here are applied to the height above the field, or, more precisely, to the height above the facility that is giving you your altimeter setting. In the present example, $20 \%$ of 1157 is about 230. Add that to 2820 to get 3050, which is the number you want to see on your altimeter during final approach. Note that this number, 3050, represents a peculiar mixture: 1663 real feet plus 1387 shrunken feet.

For better accuracy, you can use the following equation. The indicated altitude you want to see is:

$$
\begin{equation*}
A_{i}=F+\left(A_{r}-F\right)^{278.15-\lambda F} \tag{20.1}
\end{equation*}
$$

In this formula, $F$ is the facility elevation, $A_{r}$ is the true altitude you want to fly (so $A_{r}-F$ is the height above the facility, in real feet), $\lambda$ is the standard lapse rate ( ${ }^{\circ} \mathrm{C}$ per thousand feet), $T_{f}$ is the temperature at the facility, 273.15 is the conversion from Centigrade to absolute temperature (Kelvin), and $15 \mathrm{C}=288.15 \mathrm{~K}$ is the sea-level temperature of the standard atmosphere. The denominator $\left(273.15+T_{f}\right)$ is the absolute temperature observed at the facility, while the numerator $(288.15-\lambda F)$ is
what the absolute temperature would be in standard conditions.
You might want to pre-compute this for a range of temperatures, and tabulate the results. An example is shown in table 20.1. Make a row for each of the critical altitudes, not just the Minimum Descent Altitude. Then, for each flight, find the column that applies to the current conditions and pencil-in each number where it belongs on the approach plate.

| Facility Temp, ${ }^{\circ} \mathrm{C}$ | 12 | 0 | -10 | -20 | -30 | -40 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| South Sector | 6600 | 6820 | 7000 | 7220 | 7440 | 7700 |
| Northeast Sector | 5100 | 5240 | 5380 | 5540 | 5680 | 5860 |
| Northwest Sector | 4100 | 4200 | 4300 | 4400 | 4520 | 4640 |
| Procedure Turn | 4800 | 4940 | 5060 | 5200 | 5340 | 5500 |
| Crossing Outer Marker | 3600 | 3680 | 3760 | 3840 | 3940 | 4020 |
| Minimum Descent Alt | 2820 | 2860 | 2920 | 2960 | 3020 | 3080 |

Table 20.1: Saranac Lake Critical Altimeter Readings
It is dangerously easy to get complacent about the temperature compensation. You could live in New Jersey for years without needing to think about it -- but then you could fly to Saranac Lake in a couple of hours, and get a nasty surprise.

The HALT corrrection is important whenever temperatures are below standard and your height above the terrain is a small fraction of your height above the facility that gave you your altimeter setting. This can happen enroute or on approach:

- When flying over tall mountains, you might need to apply a great deal of compensation; 11,000 shrunken feet might not be enough to get you over a 10,000-foot mountain.
- When flying a localizer approach where the minimum descent altitude is 1000 feet above the facility, you might need to apply more than 100 feet of compensation. This might make the difference between crashing and not crashing.

If it's cold, add 10\%.
If it's really, really cold, add $20 \%$.

### 20.3 Prevailing Winds and Seasonal Winds

A parcel of air will have less density if it has

- a higher temperature,
- a higher dewpoint, and/or
- a lower pressure.

If a parcel of air is less dense than the surrounding air, it will be subject to an upward force. ${ }^{5}$

### 20.3.1 Primary Circulation Patterns

As everyone knows, the tropics are hotter and more humid than the polar regions. Therefore there tends to be permanently rising air at the equator, and permanently sinking air at each pole. ${ }^{6}$ This explains why equatorial regions are known for having a great deal of cloudy, rainy weather, and why the polar regions have remarkably clear skies.

You might think that the air would rise at the equator, travel to the poles at high altitude, descend at the poles, and travel back to the equator at low altitude. The actual situation is a bit more complicated, more like what is shown in figure 20.9. In each hemisphere, there are actually three giant cells of circulation. Roughly speaking, there is rising air at the equator, descending air at 25 degrees latitude, rising air at 55 degrees latitude, and descending air at the poles. This helps explain why there are great deserts near latitude 25 degrees in several parts of the world.


## Figure 20.9: Primary Circulation Cells

The three cells are named as follows: the Hadley cell (after the person who first surmised that such things existed, 250 years ago), the Ferrel cell, and the polar cell. The whole picture is called the tricellular theory. It correctly describes some interesting features of the real-world situation, but there are other features that it does not describe correctly, so it shouldn't be taken overly-seriously.

You may be wondering why there are three cells in each hemisphere, as opposed to one, or five, or some other number. The answer has to do with the size of the earth ( 24,000 miles in circumference), its speed of rotation, the thickness of the atmosphere (a few miles), the viscosity of the air, the brightness of the sun, and so forth. I don't know how to prove that three is the right answer --- so let's just take it as an observed fact.

Low pressure near 55 degrees coupled with high pressure near 25 degrees creates a force pushing the air towards the north§ in the temperate regions. This force is mostly balanced by the Coriolis force associated with motion in the perpendicular direction, namely from west to east. As shown in figure 20.10, these are the prevailing westerlies that are familiar to people who live in these areas.

According to the same logic, low pressure near the equator coupled with high pressure near 25 degrees creates a force toward the equator. This force is mostly balanced by the Coriolis force associated with motion from east to west. These are the famous trade winds, which are typically found at low latitudes in each hemisphere, as shown in figure 20.10.


Figure 20.10: Primary Prevailing Winds
In days of old, sailing-ship captains would use the trade winds to travel in one direction and use the prevailing westerlies to travel in the other direction. The regions in between, where there was sunny
weather but no prevailing wind, were named the horse latitudes. The region near the equator where there was cloudy weather and no prevailing wind was called the doldrums.

The boundaries of these great circulatory cells move with the sun. That is, they are found in more northerly positions in July and in more southerly positions in January. In certain locales, this can produce a tremendous seasonal shift in the prevailing wind, which is called a monsoon. ${ }^{\underline{7}}$

### 20.3.2 Continental / Oceanic Patterns

Now let us add a couple more facts:

1. The sun is not very effective at heating the air, especially dry air. Normally, the sun heats the surface of the planet, then the air gains heat from the surface --- partly by simple contact, and partly by absorbing energy-rich water vapor that evaporates from the surface.
2. When we change from winter to summer, solar heating warms the dry land much more quickly
 you need only heat up the top few inches of soil. To heat up the ocean, you need to heat up several feet of water.

As a consequence, in temperate latitudes, we find that in summer, the land is hotter than the ocean (other things, such as latitude, being constant), whereas in winter the land is colder than the ocean.

This dissimilar heating of land and water creates huge areas of low pressure, rising air, and cyclonic flow over the oceans in winter, along with a huge area of high pressure and descending air over Siberia. Conversely there are huge areas of high pressure, descending air, and anticyclonic flow over the oceans in summer.

These continental / oceanic patterns are superimposed on the primary circulation patterns. In some parts of the world, one or the other is dominant. In other parts of the world, there is a day-by-day struggle between them.

### 20.4 Summary

Very near the surface (where friction dominates), air flows from high pressure to low pressure, just as water flows downhill. Meanwhile, in the other $99 \%$ of the atmosphere (where Coriolis effects dominate) the motion tends to be perpendicular to the applied force. The air flows clockwise§ around a high pressure center and counterclockwise§ around a low pressure center, cold front, or warm front.

Although trying to figure out all the details of the atmosphere from first principles is definitely not worth the trouble, it is comforting to know that the main features of the wind patterns make sense. They do not arise by magic; they arise as consequences of ordinary physical processes like thermal expansion and the Coriolis effect.

If you really want to know what the winds are doing at 10,000 feet, get the latest 700 millibar constant pressure analysis chart and have a look. These charts used to be nearly impossible for generalaviation pilots to obtain, but the situation is improving. Now you can get them by computer network or fax. On a trip of any length, this is well worth the trouble when you think of the time and fuel you can save by finding a good tailwind.

A few rules of thumb: eastbound in the winter, fly high. Westbound in the winter, fly lower. In the
summer, it doesn't matter nearly as much. In general, try to keep low pressure to your left§ and high pressure to your right§.

1
The origin of the Coriolis effect is discussed in section 19.3.

The bottom box starts at sea level at both sites. We ignore the fact that OSH is actually 808 feet above sea level. The fact that the ground "sticks up" into the bottom box doesn't change the essence of the argument. This is consistent with the notion that you adjust your altimeter to read 808 (not zero) on the ground at OSH.

The idea to get you down as low as possible, to maximize your ability to get below the cloud ceiling so you can find the airport in bad weather.

It would be simpler, but less accurate, to say "hot air rises". For one thing, if all the air is hot, none of it will rise. Secondly, it is important to keep in mind that an upward force is not necessarily the same as upward motion.

Although there is, as expected, somewhat low pressure at the equator (and very low density, when you take humidity into account), there is not any noticeable high pressure at the poles. In fact, there is phenomenally low pressure at the south pole. I have no idea why this is. Sorry.

The word "monsoon" comes from an Arabic word meaning "season", hence "seasonal wind". In southern Arizona the word is properly used to describe the seasonal wind that brings rain in July. The rain is not very heavy but contrasts with rainless June. On the west coast of India, in one season the wind comes from the ocean, bringing torrential rainfall. The word properly refers to any seasonal wind, not just the rainy season, and not necessarily heavy rain. Nonexperts commonly use the word "monsoon" as shorthand for "monsoon rains" or even "torrential rain" but that's not recommended.

A similar thing happens, on a smaller scale, when we change from night to day.

## 21 Pilot-In-Command Decisionmaking

### 21.1 Decisionmaking In General

Piloting requires a range of skills:

- At one extreme are "reflexes" that involve lots of eye-hand coordination and tactics rather than strategy. There are also some basic thinking skills, such as giving answers to clearly-posed questions. These skills are relatively easy to teach, and relatively easy to evaluate.
- At the other extreme are advanced decisionmaking skills. As pilot in command, you will need to make decisions in situations where it's not even obvious that a decision needs to be made. This requires being able to notice small things as well as being able to see the big picture. It also requires not assuming that whatever procedure you used last time is going to work next
time. These skills are extremely important. They are, alas, not so easy to teach or easy to evaluate.

As an example: In a lesson, you might be asked to demonstrate short-field landing procedure. It isn't your decision, since you have been told to use short-field procedure. In contrast, suppose that years from now, you want to fly yourself to XYZ airport. Your former instructor knows the XYZ runway is very short, but do you? The instructor won't be there to tell you to use short-field procedure. What's worse, there won't even be anybody to ask you what procedure is called for. Will you be wise enough to ask the question on your own? Perhaps questions such as "should we be using short-field procedure?" and "should we be using this runway at all?" ought to be on the takeoff and landing checklists, but in light aircraft they typically aren't.

- Let's assume you know how to calculate the runway requirements using the POH. That's easy, if and when somebody asks the question.
- Let's assume you know how to use short-field technique. That's not the issue.
- The question is, will you know whether to use short-field technique, when there's nobody there to ask the question.

This sort of decisionmaking is a high-level skill. It is not well understood. Common sense is good, but this goes beyond common sense. Planning ahead is good, but this goes beyond planning. Logic is good, but this goes beyond mere syllogisms. Strategy is good, but this goes beyond the usual definition of strategy. Let's just call it decisionmaking. (Some pilots also call it headwork.)

There are innumerable situations that require good decisionmaking.
As just discussed, one aspect of decisionmaking is to recognize that a decision is needed, even though it's not in the form of a clearly-posed question. A somewhat-related aspect involves starting from a seemingly-small, seemingly-isolated fact, then seeing how it connects to the other facts you know, and working out the implications and ramifications.

As an example of ramifications: Suppose you were expecting a $20-\mathrm{knot}$ tailwind, but you've actually got a 20 -knot headwind. For starters, you have to notice this. It won't be obvious, unless you're checking arrival-time at enroute waypoints, and/or checking DME or GPS groundspeed. Then you have to work out the implications.

The obvious implication is that you're going to be late.
A slightly-less-obvious implication is that you might not have enough fuel to reach your originally-intended destination. If you have to divert, don't wait until you're low on fuel to do it. An even-less-obvious implication is that if the forecast is wrong about the winds, it's probably wrong about everything else: ceiling, visibility, temperature, precipitation, icing, turbulence, et cetera. You'd be well advised to get an updated forecast.

### 21.2 Decisionmaking during Lessons

I have adopted an instructional style designed to exercise your decisionmaking muscle. (Other instructors may have different styles. Discuss it in advance with your instructor, to avoid misunderstandings.)

### 21.2.1 Please Act As PIC During Lessons

During introductory lessons, you start out with minimal responsibilities and gradually acquire more and more. Later, in non-introductory situations, I want you to act like pilot-in-command as much as possible.

With one exception (section 21.2.3), everything I say is merely a suggestion. My suggestions aren't meant to relieve you of your responsibility as pilot-in-command. For instance, if I ask you to turn right, you remain responsible for clearing the area. Please clear the area just as you would if you were solo. Also think about whether the new heading will take us into restricted airspace or some such. If you don't want to turn right, we can discuss it.

If we are in a situation that you ought to be able to handle on your own, I will generally let you handle it. If I need to contribute, I like to use a multi-stage "escalation" process:

- Ideally, I don't need to say anything. If we are facing an energy-management challenge, you can notice it (the sooner the better) and deal with it.
- If you don't deal with it on your own, l'll start asking questions, such as: "How's your energy? Are we high and fast, or low and slow?"
- Then come more-explicit statements: "It looks like the angle from the horizon to the aim point is growing. If you don't do something we're going to overshoot the runway."
- Then it escalates to an instruction: "Go around."
- Then the instructions become more detailed: "Add power. Raise the nose. Start retracting the flaps."
- Finally: "I've got it."

Remember, being a pilot means making decisions, even during lessons (except introductory lessons). During the escalation process, I'm gradually shifting more of the decisionmaking onto my shoulders. Your goal should be to take the hints at the earliest possible stage, so that further stages of escalation are not needed.

As another illustration of the same idea, I try to avoid giving an instruction such as "go around". If I see a deer on the runway, l'll say there's a deer on the runway, and you can come to your own conclusion about going around. If we need to do some go-arounds just for practice, I might say there's a hippopotamus on the runway. You know it's not real, but I want you to pretend there's an obstruction, and come to your own conclusions about how to deal with it. Most likely you will decide to go around.

At the other extreme, if you are struggling with an unfamiliar situation, l'll just tell you how to deal with it. No hints, no escalation. There are lots of good but non-obvious techniques, and I don't expect you to re-invent them on your own.

Also note that one element of good PIC decisionmaking is knowing when to ask for help. This includes asking for clarification of an overly-vague hint.

### 21.2.2 How's Your Workload?

From time to time I will ask you "How's your workload?" There are many possible answers, including:

- Swamped. Could use some help.
- Pretty busy right now.
- Workload's not too bad right now.

If you're swamped, l'll help. If you're busy, l'll leave you alone. If you're not busy, I might strike up a conversation about strategy or tactics, or suggest an exercise.

### 21.2.3 I've Got It

If I say "I've got it", that means I am taking command of the airplane and I don't want any delay or any question about it. (We will discuss it afterward.)

Notice the important distinction:

- "How about I fly for a bit?" or "Would you like me to demonstrate that maneuver?" Those are simply questions, perhaps verging on suggestions. Those are negotiable.
- "I've got it." This is not a suggestion. This is absolutely not negotiable. This is necessary to preserve safety.


### 21.2.4 Hood Work

When you are under the hood, practicing flight by reference to instruments, you should start by telling your safety pilot (whether it's me or somebody else) "I'm delegating the traffic-spotting to you". The safety pilot should give you a readback on this, saying something like "I accept the delegation". You should insist on this.

You remain pilot-in-command and you even retain a share of the responsibility for traffic separation. Before turning (except small shallow turns), ask "Clear right?" or "Clear left?"; don't assume your safety-pilot has pre-cleared all turns.

When practicing an instrument approach, I will tell you if/when we break out of the simulated clouds. If in doubt, you can ask whether we have broken out. This is another decisionmaking exercise. If we reach the missed-approach point or decision height and haven't broken out, do not expect me to say anything at this point. I want you to decide on your own when it's time to begin the missed-approach procedure.

If I say "you've got the approach lights" it means you haven't entirely broken out of the simulated clouds, but you have the option of continuing the approach in accordance with FAR 91.175(c)3(i).

### 21.3 Layers of Safety

One of the standard ways to achieve a high level of safety is to use a layered approach: layers and layers of backups and crosschecks.

For example, before takeoff, always check the fuel level by looking in the tanks. If you can't accurately judge the fuel level by eye, use a pipette to measure it. Then see what the cockpit fuel gauges are reading. Crosscheck the two types of measurement. If they disagree, you've got a problem.

Similarly, during the course of the flight, you have two ways of estimating how much fuel remains: (a) Start with what you had at takeoff, and decrement it according to the expected fuel-burn rate, and (b) look at the gauges. Method (a) will fool you if there is a leak or other problem in the fuel system, and method (b) will fool you if the gauge is stuck, but the chance of both problems happening at the same time is remote. (The third layer of safety is to make a forced landing, but you hope it doesn't come to that.)

Don't lightly give away layers of safety. For example, if you don't do a magneto check before each flight, it's just a matter of time before one mag fails. You won't notice this, especially if it is the right mag (since only the left mag is used for starting). Then it's just a matter of time before the other mag fails in flight. You will notice it then, because the engine will suddenly quit.

The notion of layers of safety applies to many aspects of flying:

- Having two magnetos doesn't make the engine twice as reliable; it makes it thousands of times more reliable.
- Pipetting the tanks and keeping an eye on the fuel gauges is thousands of times more reliable than either one separately.
- As discussed in section 12.7.4 there is a long list of cues you can use to make sure you aren't landing with a tailwind and/or excessive airspeed. Sometimes some cues will be useless or worse: perhaps the windsock is not visible from pattern altitude, and perhaps the previous airplane used the wrong runway. And perhaps you will occasionally overlook one or two cues. But that still leaves many, many cues that will keep you out of trouble.
- You have multiple sources of navigation information (dead reckoning, pilotage, VOR, GPS, ATC radar) which can be cross-checked against each other.

You should pay attention to anything that peels away one or more layers of safety. Keep track of how many layers remain.

If one magneto fails, park the airplane until it is fixed!
If the fuel gauges cannot be trusted, park the airplane until it is fixed! Do not rely on clock-and-dipstick alone, or on the gauges alone.

### 21.4 Example: Obstacle Clearance

In other publications, obstacle clearance is commonly discussed under the heading "controlled flight into terrain" (CFIT). The term obstacle clearance is preferable, partly because it puts a more positive spin on things: it is better to talk about your obstacle clearance successes than your CFIT failures. Also, the CFIT statistics are misnamed because they include collisions with trees, man-made structures, bodies of water, etc. that you might not have thought of as "terrain". They also include taxiing into potholes and other things that you might not have thought of as "flight".

We ought to pay serious attention to the obstacle clearance issue, because statistics show a surprisingly large number of accidents where a perfectly good aircraft collides with an obstacle. You would think such accidents would be entirely preventable, so even one occurrence is far too many.

Obstacles can be a factor during any phase of flight, including departure, enroute, or approach. A typical accident scenario goes something like this: At night (or in hazy weather), at an unfamiliar airport, the pilots crash into power lines or into a hillside.

Let's analyze this scenario using our notion of layers of safety. Let's ask what "caused" this accident.

- Did they crash "because" of the obstacles? If they had been flying somewhere with more benign topography, they wouldn't have gotten into trouble.
- Did they crash "because" they made a wrong turn? Presumably every airport has some path that airplanes can safely follow on takeoff. Perhaps they weren't familiar with the correct procedure, or perhaps they just neglected to follow the correct procedure.
- Did they crash "because" of darkness or hazy weather? If it had been daytime in clear weather, they would have seen the approaching obstacle in time to turn away.

We say this accident had at least three causative factors. Each of the factors was "a" contributory cause of the accident, but none was "the" sole cause of the accident.

Multi-factor situations like this can be a challenge to your decisionmaking skills. Section 21.3 says you should not lightly give away layers of safety. But what does "lightly" mean? Sometimes there are good reasons for accepting some risk. Sometimes it's OK to fly at night, or in hazy weather. Sometimes it's OK to fly to an unfamiliar field. But don't get too complacent. If you get complacent about each risk factor separately you can get into big trouble if/when multiple risk factors gang up on you.

Anything that involves operating at low altitudes peels off one or two layers of safety. In addition to ordinary approaches and departures, there are many examples including patrol, photo work, crop dusting, scud running (i.e. flying at low altitudes to avoid clouds), buzzing (i.e. flying at low altitudes to show off), and mountain flying.

The departure phase and approach phase account for a huge proportion of the obstacle clearance problems. You need to worry about this even in regions that are not considered mountainous. A modest hill or a modest structure can be a serious threat if it's near a runway.

VFR at an unfamilar field at night (or in hazy weather) is particularly risky, as discussed in section 12.1.3 and section 13.7.5.

In the enroute phase, the primary obstacle-clearance technique is to choose a suitable route and a suitable altitude, as discussed in section 14.8. A good secondary technique, to reduce the chance of mistakes, is to get radar advisories. The ATC computers know the minimum safe enroute altitude in each sector, and will sound an alarm if you get too low. Similarly, some fancy RNAV units now contain obstacle-clearance data and will give you warning of approaching threats. Another thing that may be of some help is an altitude alerter. This is a simple, cheap instrument. You tell it what you have chosen as your intended altitude, and it will beep if you inadvertently drift above or below that. Alas, this won't help much if you punch an unsuitable number into the instrument (due to bad planning or whatever) and it won't help if you are trying to fly through a mountain pass and get offcourse horizontally.

### 21.5 Flow Pattern

During the preflight check, you should walk around the airplane and check everything that you come to, in order. This is an example of using the flow pattern.

After examining things according to the flow pattern, you should run the checklist to see what you overlooked.

Checklists are good. Flow patterns are good. Neither one is a substitute for the other; instead, each is a backup for the other. Using them both is much, much better than using either one alone.

There are many situations where you can use a flow pattern, including:

- Preflight walk-around.
- Pre-takeoff instrument check.
- Enroute instrument check.
- Engine failure / restart (section 15.1.1).
- Et cetera.


### 21.6 Checklists

If you don't use a written checklist, it is just a matter of time until you forget something.
There are some checklists that you should commit to memory, such as rejected takeoff, spins, fire in flight, go-around, and possibly others, depending on how complex your airplane is. But even these should not be entrusted to long-term memory. Short-term memory is better than long-term memory, so refresh your memory at frequent intervals. An excellent method is to recite the checklist out loud, while somebody else checks your version against the written version.

I know a lot of pilots who fastidiously use a written checklist for preflight, but rely on memory for the approach and landing checklists. It is fairly easy to see how people fall into this trap: During preflight you are not strapped into your seat, and you are not busy flying the airplane. You can always take a minute to find the POH and read through it. In contrast, when you are setting up for a landing, the book is likely to be somewhere hard to reach and you're too busy to do much reading anyway.

Therefore, here are some constructive suggestions. Pick the one(s) you like best:

- Make a pocket checklist. Print up the checklists you are going to need during flight and fold them in such a way that they fit in a pocket. During preflight, put the list in a convenient pocket.
- If you habitually use a lap desk, tape a copy of the in-flight checklists to the lap desk itself.
- If space can be found, glue a copy of the in-flight checklists to the instrument panel.

The pocket checklist should include the approach, landing, and go-around checklists, as discussed in section 12.1.

If you fly more than one airplane, make sure you have an appropriate pocket checklist or lap-desk checklist for each of them. As you progress in your pilot career, you will be flying progressively more complex aircraft, and if you persist in using the same old checklist you will get into trouble some day. Some aircraft have retractable landing gear; some don't. Some aircraft have cowl flaps; some don't. Some aircraft require using carburetor heat; some don't. Some aircraft require switching on the electric fuel pump for landing; others forbid it.

Discipline yourself to pay attention to the checklist. Don't just keep it in your pocket as a good-luck charm.

My pocket checklist also includes an enroute checklist, which is only three words long: "indications, configuration, location". That is shorthand for the following:

Indications: Go left-to-right across the panel and check everything, including the gauges that aren't part of your ordinary moment-to-moment scan: Fuel level, engine instruments, et cetera. Check that the directional gyro is aligned with the compass.
Configuration: See if the fuel/air mixture is appropriate for this phase of flight. Make sure you're not flying cross-country with the flaps extended. On more complex airplanes, there are more things to check: landing gear, cowl flaps, speed brakes, et cetera.

Location: Where are we? Where's the nearest airport? Mark this location on the chart, along with the time, as discussed in section 14.2.

### 21.7 Personal Minimums

All too often, pilots get into risky situations without even realizing it. They don't consciously decide to run a risk. They just take off on a supposedly routine flight, and by the time they notice a problem it is already too late to solve the problem.

In theory, you can avoid such problems by paying meticulous attention to "everything". But in reality, it is unreasonable to expect people to be super-meticulous all the time. The trick is to be aware of what's routine and what's not. It helps to have a clear-cut set of personal minimums.

Personal minimums are distinct from regulatory minimums. For instance, the regulations might permit a pilot with little experience to fly an unfamiliar aircraft on a maximum-range mission over water at night in moderate turbulence, then land in a 25-knot crosswind on an unfamiliar narrow obstructed unlighted runway, having had little sleep the night before ... but I don't recommend it.

Write down your personal minimums in the form of a checklist. Review the list before each proposed flight. If you are within the limitations, fine. If the proposed mission is slightly outside the envelope in one or two aspects, you might want to go ahead with it anyway -- provided you are super-meticulous. The checklist is warning you that this flight is not routine.

Issues to consider include:

1. At least $\qquad$ runway length for $\qquad$ airplane for density altitude below 2000 feet.
2. At least $\qquad$ runway length for $\qquad$ airplane for density altitude between 2000 and 5000 feet.
3. At least $\qquad$ runway width.
4. At most $\qquad$ knots gust component along the runway.
5. At most $\qquad$ knots across the runway, including gusts.
6. At least $\qquad$ fuel reserves.
7. Ceiling $\qquad$ and visibility $\qquad$ for VFR.
8. Ceiling $\qquad$ and visibility $\qquad$ at destination for IFR.
9. Survival equipment for flight over wilderness.
10. Survival equipment for flight beyond gliding range from dry land.
11. At least $\qquad$ hours experience in this make \& model.
12. At most $\qquad$ pressure altitude without oxygen mask.
13. Physiological issues: "I-M-SAFE"

- Illness?
- Medication?
- Stress?
- Alcohol?
- Fatigue?
- Eating?
14.Turbulence?

15. Dark VFR?
16. Mountain flying? Bush flying? Obstructions enroute? Obstructions near the airports to be used? Low-altitude operations?
17. In-flight "how-goes-it" checkpoints and "proceed/divert" checkpoints.

There are lots of other things you can legally do with just a private pilot certificate that are, alas, not covered in the basic private-pilot training. Examples include

- If you're flying on a moonless night over unlighted areas, it can be a challenge to keep the airplane right-side-up. This requires instrument-flying skills far beyond what is required for the private-pilot checkride. (Night VFR is prohibited in many countries, but not all.)
- Specialized skills are required for mountain flying, bush flying, low-level patrol, crop dusting, aerobatics, formation flying, et cetera.
- Beware of the obstacle-clearance issues listed in section 21.4.
- The first time you fly into a big, busy place like O'Hare, you might want to take along an instructor or at least a copilot who's been there before.
- Hand-propping (i.e. starting the engine by pulling on the propeller) is potentially very dangerous. You may have seen it done in some old Hollywood movie, but that doesn't mean you're qualified to do it. Don't try it unless you've got a good reason and have been thoroughly trained on the procedure.
- If you routinely fly solo out of a short field, it doesn't prove you can depart from there carrying three large passengers and their luggage. Maybe you can, maybe you can't, but it's not routine, so you need to get out the book and redo the weight \& balance calculations, the performance calculations, et cetera.

These are just a few examples out of many. It is your responsibility to recognize when a situation is outside the envelope of your training and experience. It is your responsibility to acquire whatever skills are required.

Don't try to impose your personal minimums on the pilot next door. Yours will be too strict in some aspects and too lax in others. Your personal minimums are designed by you, for you. That's why they're called "personal" minimums.

The idea of personal minimums applies during the whole flight, not just during preflight. In some cases you should establish specific personal checkpoints for a specific flight. For instance, if you're flying into questionable weather, pick a specific checkpoint where you will get an updated weather report and decide whether to divert or not. Similarly, if it's a maximum-range mission, establish checkpoints along the way, where you will re-evaluate the headwinds, fuel quantity, et cetera. The idea is to notice early if Plan A isn't working, so you can execute Plan B while there's still time.

### 21.8 Skepticism; Crisp Execution of Plan B

You need to notice things. That means more than just seeing things; you need to appreciate the significance of what you are seeing.

Skepticism means, among other things, not assuming that the way things are is the way things should be. For example:

- Suppose that during the preflight check you see a red light on one wingtip and a green light on the other wingtip. Are you sure you would notice if they were interchanged, so that both wingtips had the wrong color?
- Suppose you see that the tow-bar is lying loose in the back of the airplane. You could just leave it that way, on the theory that people have flown the airplane hundreds of times in that condition without getting into trouble. But suppose you get into an unusual attitude or a minor crash; you don't want the tow-bar to come whizzing toward you like a spear. Leaving it loose is
needlessly throwing away one of your layers of safety. Anything that can't be stowed securely should be left on the ground until you get back.
- Suppose you find a puddle of oil underneath the engine. You must be skeptical about where it came from. l've seen puddles on about 20 occasions. Usually it's just because some klutz spilled oil while trying to add it to the crankcase. But on one occasion it was due to a moderately serious leak in the engine, and on another occasion it was due to a very serious crack in a cylinder, the sort of crack that will get rapidly worse during flight. The only way to know for sure is to take off the cowling, mop up the mess, run the engine for a few minutes, and check for leakage. That's a lot of bother, but it's preferable to risking in-flight engine failure.

You want to be properly skeptical without being paranoid. Nothing in this world is perfect. If you cancel the flight whenever the airplane is not perfect, you'll never go flying. Judgement is required.

Also: Piloting requires flexible thinking. Do not think you can plan a flight and then fly it exactly as planned.

- First of all, you have to have a Plan B. (See section 21.3.)
- Secondly, you need to promptly recognize when Plan A has gone to pot. (See also section 21.7.)
- If Plan A isn't working, don't persevere with Plan A! We need crisp execution of Plan B.

The very first takeoff on my private pilot checkride was supposed to be a simulated soft-field takeoff. That requires, among other things, not stopping after leaving the run-up area, lest we sink into the simulated mud. I was nervous. I wanted to make a good first impression.

We were cleared for takeoff on runway 15 Left. Another aircraft was cleared to land on runway 15 Right. As I turned onto the runway, I heard Tower yelling at the other guy, pointedly reminding him to land on the right. I didn't hear any response. I had no idea whether the other aircraft was near or far; all I could do was try to imagine what could cause Tower to say such things. I imagined that the other pilot was planning on landing on the left and was persevering with his Plan A even though he'd been cleared to do something else.

I didn't like what I was imagining, so I pulled the throttle to idle and stomped on the brakes. I also keyed the transmitter and said "Tower, Two-Four-Kilo is gonna hold our position for a moment". Now I was really nervous. I had planned to comply with all ATC clearances, such as takeoff clearance. I had also planned to comply with the examiner's request for soft-field procedure. But there I was, stopped on the runway. My Plan A was in shambles.

I had no idea whether holding my position would make things better or worse. I figured it was a 50/50 chance. The deciding factor was not the odds but the relative payoff: if there's going to be a collision, l'd rather have a collision on the ground than in the air.

A moment later, the other aircraft flew right over top of us, about 10 feet up. My Plan B was starting to look pretty good.

Tower said "Two-Four-Kilo, advise when you're ready for departure". I said "as soon as that guy is out of our hair, we're ready". Tower acted like it was no big deal, and just said "Two-Four-Kilo, runway 15 Left, cleared for takeoff'.

The examiner sat there with his poker-face on. I couldn't tell whether or not he approved of what l'd
done. Looking back with the benefit of years of PIC experience, I'm quite sure I did the right thing, and I'm quite sure the examiner and the tower controller were glad that I deviated from their instructions.

Still, it is worth remembering that at the time, I was uncertain about this. I found the decision difficult and stressful. People aren't born with advanced decisionmaking skills. Training is needed.

### 21.9 Leadership and Resource Management

Be smart about using all the resources available to you.
If you have a copilot, that's an important resource. You can delegate certain tasks to your copilot.

- A very useful technique for expediting a flight is to let one pilot do the pre-flight walkaround while the other deals with weather, flight plans, et cetera.
- In busy airspace, it works well to let one pilot work the radios while the other does everything else.
- Copilots and even passengers can help spot traffic.
- There are many other examples of effective collaboration. Entire books have been written on the subject.

On the other hand, make sure you don't get into a situation where two pilots are worse than one. This can happen more easily than you might think:

- l've seen situations where each pilot assumed the other would take care of something, but it never got taken care of.
- l've even seen the following: It was time to toggle to a new frequency. One pilot pushed the "toggle" button. The other pilot pushed it also. As a result, the frequency was not what either pilot was expecting.

To prevent such problems, make sure there is agreement about who is pilot-in-command. If you are PIC, you retain final authority and responsibility for everything. If you delegate something, make sure the delegation is understood and carried out. If you are second-in-command, make sure the PIC knows what you've done and not done.

Learn from the story of Eastern Airlines flight 401. They crashed an airliner into the swamp partly because all three pilots were preoccupied with debugging a landing-gear indicator light. (Of course this is only part of the story.) Whether you have one pilot on board or three, don't let a small problem interfere with your basic responsibility to fly the airplane. That includes maintaining a safe altitude, seeing and avoiding other traffic, et cetera. If you have more than one pilot, let one fly the airplane while another debugs the problem.

Fly the airplane.

Resource management is commonly called "cockpit" resource management (CRM), but it should include resources outside the cockpit, notably Flight Service and ATC.

A related issue is cockpit leadership. If you are PIC, don't act like Captain Bligh. Encourage your crew members to speak up if they see anything questionable. Keep them informed as to your intentions, so
that they will be better able to notice if something unintended is happening. If you are a crewmember, it is your duty to speak up if you see something amiss, no matter how surly the PIC is.

### 21.10 Learn from the Experience of Others

When pilots get together, they often trade stories about accidents or near-accidents. Non-pilots are sometimes shocked; they think it's ghoulish. But that's not the point at all. The point is to learn what led up to the problem, and what can be done to prevent a recurrence.

You should study the accident statistics, so you know what are the big worries and what are the relatively minor worries. There are many sources of such information, including:

- Various pilot-oriented magazines feature every month a new anecdote from some pilot who learned something the hard way.
- The National Transportation Safety Board keeps records on all accidents and incidents. These are available on their web site.
- NASA's Aviation Safety Reporting System collects reports on untoward events, even those that don't result in accidents. Every month they publish a discussion called Callback which is available from the NASA website and otherwise.
- The AOPA Air Safety Foundation puts out the annual Nall Report summarizing statistics from multiple sources, taking a general-aviation point of view. This is available from their web site and otherwise.
- There are many published books containing aviation biographies or collected stories.


### 21.11 Try to Outdo Yourself

Part of the romance of aviation is to do everything better than necessary. If the runway is 50 feet wide and the airplane's wheelbase is 10 feet wide, it is technically possible to land on the left half or the right half of the runway. But everybody tries to land exactly on the centerline. If you were off by one foot last time, try to be off by half a foot next time.

Safety is not directly affected by your best performance, or even your average performance. What matters, directly, is your worst-ever performance. This is called the minimax principle: make sure your worst-case performance is good enough. This partly involves skill, but largely involves using judgement to stay out of situations (distractions, fatigue, bad weather, etc.) that might cause your performance to be significantly worse than usual.

High standards contribute indirectly to safety in the following way: If your usual tolerances are tight enough, then even on the super-rare occasions when your performance is ten times worse than usual, you will still have a wide margin of safety.

